Econ 303 - Intermediate Microeconomic Theory
College of William & Mary
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Problem Set 3

This problem set will be graded and is due by **5pm** on **Friday, March 1st** in my mailbox in the economics department. You may turn in problem sets early by putting them in my mailbox in the economics department or by dropping them off in lecture. No late problem sets will be accepted. You are welcome to work in groups. If working in a group, everyone in the group must still submit an individual problem set.

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1. **Deriving Market Demand** There are four individuals, Aaron, Bob, Carl and David, who buy widgets. Their individual demands are given by the following inverse demand functions:

$$P(x_A) = 10 - x_A \quad (1)$$

$$P(x_B) = 20 - 2x_B \quad (2)$$

$$P(x_C) = 20 - x_C \quad (3)$$

$$P(x_D) = 15 - 2x_D \quad (4)$$

   (a) Derive an expression for the market demand for widgets (you can assume that Aaron, Bob, Carl and David are the only people buying widgets). Graph this market demand with price on the vertical axis and widgets on the horizontal axis.

   (b) Along which segment of the market demand curve is demand most elastic? Along which segment is demand most inelastic?

   (c) Suppose that the market supply of widgets at a given price is $S(P) = -6 + \frac{3}{2}P$. What is the equilibrium price and quantity? Which consumers are buying positive amounts of the good?

   (d) So far, all of the kinked market demand curves we have looked the individual segments of the demand curve get flatter as we move to the right. Suppose that we have multiple consumers, all with linear individual demand curves. Is it possible that the market demand curve for this group of consumers gets steeper as we move to the right? Why or why not?

2. **Deadweight Loss and the Size of Taxes**

   (a) Suppose that the demand for packs of cigarettes is given by $D(P) = 180 - 40P$ where P is the price of a pack of cigarettes. The supply of cigarettes is given by $S(P) = -20 + 10P$. Solve for the equilibrium price and quantity of packs of cigarettes.

   (b) Now suppose that the government adds a tax of $\$t$ per pack of cigarettes to be paid by consumers. What is the new equilibrium quantity of packs of cigarettes? What is the after-tax price paid by consumers? What is the price received by sellers? Does the proportion of the tax burden payed by consumers depend on the size of the tax? (Your answers may have t in them.)

   (c) Find an expression for tax revenue as a function of t. Graph tax revenue as a function of t. If the government wanted to maximize tax revenues, what tax would they choose?

   (d) Find an expression for deadweight loss as a function of t. Graph deadweight loss as a function of t.

   (e) Suppose the government cares about getting more tax revenue but also cares about keeping the deadweight loss from the tax relatively small. Consider the following objective function for the government:

$$\max_t f(\theta, TR, DWL) \tag{5}$$

   where TR is tax revenue, DWL is deadweight loss and θ is a number between zero and one that captures how much weight the government places on generating tax revenue relative to creating deadweight loss. If θ equals one, the government cares only about tax revenue and the objective function should be reduced to simply being $\max_t TR$. If θ equals zero, the government cares only about minimizing deadweight loss and the objective function should be reduced to simply being $\min_t DWL$ (which is the same as $\max_t -DWL$). Come up with a function $f(\theta, TR, DWL)$ that satisfies these properties. Specifically, your function should do the following:

   - $f(\theta, TR, DWL)$ should increase as TR increases.
   - $f(\theta, TR, DWL)$ should decrease as DWL increases.
   - $f(\theta, TR, DWL)$ should reduce to just a function of TR when θ equals one.
   - $f(\theta, TR, DWL)$ should reduce to just a function of DWL when θ equals zero.

   (f) Use your function from the previous part to find the optimal tax t as a function of θ, the weight the government places on tax revenue relative to deadweight loss.

3. **Taxes and Perfectly (In)elastic Supply**

The market demand for clothing is given by:

$$D(p) = 500 - 5p \tag{6}$$

The City of Williamsburg decides to introduce a new sales tax on clothing of 10%. Note that this is a value tax (not a quantity tax).

(a) Suppose that clothing supply is perfectly inelastic: whatever the price of clothing is, stores will always sell 400 units. What will the equilibrium price and quantity of clothing be before the new sales tax?

(b) Given the description of the supply from part (a), what will the equilibrium quantity be after the sales tax is introduced? What will be the new price paid by consumers and the new price received by sellers? What is the deadweight loss generated by the sales tax?

(c) Suppose that instead of clothing supply being perfectly inelastic, it is perfectly elastic: stores will sell any quantity of clothing but they will always sell it at a price of $20 per unit. Given this description of supply, what is the equilibrium price and quantity of clothing sold before the tax?

(d) Given the description of the supply from part (c), what will the equilibrium quantity be after the sales tax is introduced? What will be the new price paid by consumers and the new price received by sellers? What is the deadweight loss generated by the sales tax?

(e) Assume the government needs to raise T dollars of tax revenue. The government is deciding to place a tax on either food which has a fairly inelastic demand or on concert and movie tickets which have a fairly elastic demand. Given your answers to parts (a) through (d) of this question and your answers to Question 2, which of these two taxes should the government pursue? You should explain your answer with the help of graphs of the effects of the taxes in each market.

4. **Production Functions**

   For the first two production functions given below, derive expressions for the marginal product of x, the marginal product of y and the technical rate of substitution, graph three isoquants and determine whether the technology exhibits increasing, constant or decreasing returns to scale. For the last production function, you only need to graph the isoquants and determine whether the technology exhibits increasing, constant or decreasing returns to scale.

   (a) $f(x,y) = 5x^{\frac{3}{4}}y^{\frac{3}{4}}$

   (b) $f(x,y) = 2x + 3y^{\frac{1}{2}}$

   (c) $f(x,y) = \min(x, 3y)$