Econ 303 - Intermediate Microeconomic Theory
College of William and Mary May 8, 2013
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Final Exam - Solutions

You have until 3:30pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can use fractions of units of inputs, produce fractions of units of output and charge non-integer prices (so a firm could use 28.6 units of input to produce 82.4 units and sell at a price of $5.325 per unit). Remember to put your name on the exam. Good luck!

Name: **ID Number:**

1. (20 points) A monopolist has no fixed costs and constant marginal costs. The firm faces a linear, downward sloping demand curve. Determine how the following three strategies would rank in terms of profits: standard monopoly pricing, using a two-part tariff, using first degree price discrimination. Use three graphs, one for each strategy showing the total profits, and a written explanation to justify your answer.

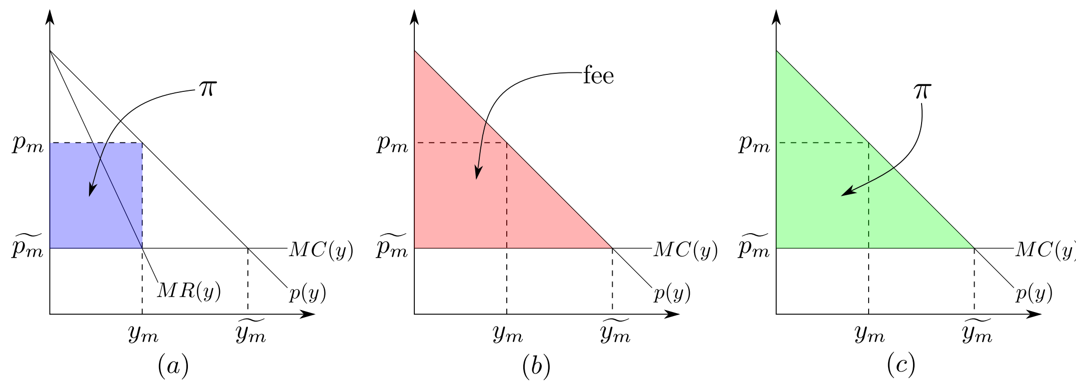
   If the monopolist uses standard monopoly pricing, it will produce at the quantity where marginal revenue equals marginal cost. The marginal revenue curve will be a straight line twice as steep as the demand curve. The marginal cost curve will be a horizontal line given that marginal costs are constant. This quantity is shown on graph (a) below, labelled as y_m. The firm will charge the price consumers are willing to pay at that quantity, p_m. On each unit, the firm will make profits equal to the price minus the marginal costs. Adding up these profits across all of the units produced gives us total profits of $(p_m - MC)y_m$, the area shaded in blue on graph (a).

   If the monopolist uses a two part tariff, it will produce any unit for which the marginal benefit exceeds the marginal cost and then capture the consumer surplus on each unit by adding it to the fee. This will lead the monopolist to produce all the way up to the point where $MC(y)$ intersects the demand curve, $\widetilde{y_m}$ on graph (b). At this point, the price consumers are willing to pay, $\widetilde{p_m}$, will just cover the firm's costs on each unit. However, the firm will charge a fee equal to the consumer surplus at that price and quantity. This fee, shown in red on graph (b), will be the profits made by the firm (it is breaking even on its sales).

   If the monopolist uses first degree price discrimination, it will charge the consumer's full marginal benefit on each unit. Therefore the profits the firm will make on each unit will be the difference between the demand curve (giving the consumer's marginal benefit) and the marginal cost curve (the firm's costs on that unit). The firm will sell any unit for which this difference is positive, leading the firm to produce all the way up to $\widetilde{p_m}$. Summing up the profits across all of these units gives the total profits of the firm, shown as the green shaded area on graph (c).

   From the graphs, it is clear that both first degree price discrimination and a two part tariff achieve the same level of profits for the firm. In both cases they provide

$\widetilde{y_m}$ units and earn profits equal to the area between the demand curve and the marginal cost curve up to that quantity. Standard monopoly pricing produces a smaller level of profits (the rectangular area on graph (a) is a subset of the triangular area on graphs (b) and (c)).



2. (25 points) There is only one dry cleaning business in Williamsburg. The dry cleaner has the following total cost and marginal cost functions:

$$C(A) = \frac{2}{5}A^2 \tag{1}$$

$$MC(A) = \frac{4}{5}A \tag{2}$$

where A is the total number of articles of clothing cleaned. Demand for dry cleaning in Williamsburg is given by the following demand function:

$$D(p) = 200 - 10p \tag{3}$$

where $D(p)$ is the number of articles of clothing people will get cleaned if the price per article is p.

(a) Assuming that the dry cleaner cannot use any forms of price discrimination, what price will the dry cleaner charge and what profits will the dry cleaner make?

The dry cleaner will maximizes profits by setting marginal revenue equal to marginal cost. For every unit up to this point, marginal revenue would exceed marginal cost meaning that profits would increase with an increase in output. For every unit after this point, marginal costs would exceed marginal revenue, suggesting that the last unit produced reduced profits and output should be lowered. To find this point, we first need to get the marginal revenue function. Before we can do this, it is useful to rearrange the demand function to get the inverse demand function:

$$D(p) = 200 - 10p$$

$$y = 200 - 10p(y)$$

$$10p(y) = 200 - y$$

$$p(y) = 20 - \frac{1}{10}y$$

Now that we have the inverse demand function, we can right out revenue is a function of y and then take the derivative to get marginal revenue:

$$R(y) = p(y) \cdot y$$

$$R(y) = (20 - \frac{1}{10}y) \cdot y$$

$$R(y) = 20y - \frac{1}{10}y^2$$

$$MR(y) = \frac{dR(y)}{dy}$$

$$MR(y) = 20 - \frac{1}{5}y$$

Setting this marginal revenue function equal to the marginal cost function will give us the monopoly quantity:

$$MR(y_m) = MC(y_m)$$
$$20 - \frac{1}{5}y_m = \frac{4}{5}y_m$$
$$y_m = 20$$

Plugging this quantity back into the inverse demand function will give us the price the monopoly will charge:

$$p(20) = 20 - \frac{1}{10} \cdot 20$$
$$p = 18$$

Finally, monopoly profits will be equal to revenues minus costs:

$$\pi = R(20) - C(20)$$
$$\pi = (20 \cdot 20 - \frac{1}{10} \cdot 20^2) - \frac{2}{5} \cdot 20^2$$
$$\pi = 200$$

(b) What is the socially efficient level of dry cleaning, A_{eff}?

It is socially efficient to produce every unit for which marginal benefit exceeds marginal cost. So the efficient quantity will be where the marginal cost curve intersects the demand curve:

$$MC(y_{eff}) = p(y_{eff})$$
$$\frac{4}{5}y_{eff} = 20 - \frac{1}{10}y_{eff}$$
$$\frac{9}{10}y_{eff} = 20$$
$$y_{eff} = \frac{200}{9}$$

(c) Calculate the deadweight loss generated by the dry cleaner acting as a monopolist.

The deadweight loss will be equal to the total surplus on all of the units between y_m and y_{eff}. This is the triangular area between the marginal cost curve and the demand curve between those two quantities:

$$DWL = \frac{1}{2}(p(y_m) - MC(y_m))(y_{eff} - y_m)$$
$$DWL = \frac{1}{2}(18 - \frac{4}{5} \cdot 20)(\frac{200}{9} - 20)$$
$$DWL = \frac{20}{9}$$

(d) Suppose that Williamsburg decides to force the dry cleaner to provide the efficient level of dry cleaning. Would the dry cleaner agree to do this or would the dry cleaner rather go out of business? Be certain to fully justify your answer.

The dry cleaner will still be willing to stay in business if it is making positive profits. If the dry cleaner would make negative profits at the efficient quantity, it would rather go out of business (and earn zero profits). So we simply need to check what the profits are at the efficient quantity:

$$\pi(y) = R(y) - C(y)$$

$$\pi(\frac{200}{9}) = \left(20 \cdot \frac{200}{9} - \frac{1}{10} \cdot \left(\frac{200}{9}\right)^2\right) - \frac{2}{5} \cdot \left(\frac{200}{9}\right)^2$$

$$\pi(\frac{200}{9}) \approx 197.5$$

So the profits are positive. The dry cleaner would continue to operate if forced to produce the efficient quantity.

3. (20 points) There are three identical farmers (A, B and C) that all sell zucchini at the farmers market. Each farmer decides how many zucchini he will bring to the farmers market. The market price for zucchini is then determined by the price consumers are willing to pay for the total amount of zucchini available ($Z_A + Z_B + Z_C$). The inverse market demand curve for zucchini is given by:

$$p(Z) = 6 - \frac{1}{90}Z \tag{4}$$

Each farmer has no fixed costs and constant marginal costs equal to \$2.

(a) Write down an equation giving farmer A's profits as a function of farmer A's number of zucchini and the number of zucchini brought by farmers B and C ($\pi_A(Z_A, Z_B, Z_C)$).

Farmer A's profits are simply his revenues minus his costs. The tricky part here is that his revenues depend on Z_B and Z_C because the price will depend on total quantity, not just farmer A's quantity:

$$\pi_A(Z_A, Z_B, Z_C) = p(Z_A + Z_B + Z_C) \cdot Z_A - C_A(Z_A)$$

$$\pi_A(Z_A, Z_B, Z_C) = (6 - \frac{1}{90}Z_A - \frac{1}{90}Z_B - \frac{1}{90}Z_C)Z_A - 2Z_A$$

$$\pi_A(Z_A, Z_B, Z_C) = 4Z_A - \frac{1}{90}Z_A^2 - \frac{1}{90}(Z_B + Z_C)Z_A$$

(b) Given your profit function in part (a), find farmer A's optimal number of zucchini as a function of the number of zucchini being brought by farmers B and C ($Z_A(Z_B, Z_C)$).

The profit maximizing quantity Z_A will be where the derivative of the π_A with respect to Z_A is equal to zero. If the derivative were positive, profits could be increased by increasing Z_A. If the derivative were negative, profits would increase by decreasing Z_A. So the optimal Z_A solves:

$$\frac{d\pi_A}{dZ_A} = 0$$

$$4 - \frac{2}{90}Z_A - \frac{1}{90}(Z_B + Z_C) = 0$$

$$\frac{2}{90}Z_A = 4 - \frac{1}{90}(Z_B + Z_C)$$

$$Z_A = 180 - \frac{1}{2}(Z_B + Z_C)$$

$$Z_A(Z_B, Z_C) = 180 - \frac{1}{2}(Z_B + Z_C)$$

This is farmer A's best response to the levels of zucchini brought by farmer B and farmer C.

(c) Using the best response function you found for farmer A in part (b), find the equilibrium number of zucchini brought by each farmer. (Hint: Recognizing that all three farmers are identical makes this much easier.)

We could solve for the equilibrium by finding each farmer's best response function and then finding the values of Z_A, Z_B and Z_C that solve all three equations. This amounts to solving a system of three equations and would lead us to the correct answer. However, we can make our lives easier by recognizing that all three firms are identical, so in equilibrium we would expect all three firms to produce the same amount. Therefore in equilibrum:

$$Z_A^* = Z_B^* = Z_C^*$$

This gives us a much simpler way to find the equilibrium quantities. We can simply use farmer A's best response function and replace Z_B and Z_C with Z_A:

$$Z_A^* = 180 - \frac{1}{2}(Z_A^* + Z_A^*)$$

$$2Z_A^* = 180$$

$$Z_A^* = 90$$

So the equilibrium quantity for each individual farmer will be 90 zucchini making the total number of zucchini supplied 270.

(d) What is the efficient number of zucchini? Is this the same as the total number of zucchini provided in equilibrium?

The efficient number of zucchini will be where price is equal to marginal cost:

$$p(Z) = MC$$

$$6 - \frac{1}{90}Z = 2$$

$$\frac{1}{90}Z = 4$$

$$Z = 360$$

So the efficient total quantity is 360 zucchini. However, the equilibrium quantity is only 270. There is an inefficiently low number of zucchini being brought to the market. If we were to increase the number of farmers, the equilibrium quantity would get closer and closer to the efficient quantity.

4. (15 points) Two roomates, Alex and Bob, are deciding how much to spend on the couch for their living room. They will share the couch and each roommate's enjoyment of the couch is independent of how much the other roomate uses the couch. Alex has a posivite but diminishing marginal benefit from couch quality given by:

$$MB_A(Q) = 200 - Q \tag{5}$$

where Q is the overall couch quality. Bob also has a positive and diminishing marginal benefit from couch quality given by:

$$MB_B(Q) = 100 - \frac{1}{2}Q \tag{6}$$

Higher quality couches cost more money. For every increase in couch quality by one unit, the price of a couch goes up \$5.

(a) If Alex were buying a couch for himself, how much will he spend on the couch?

Alex would pay for any unit of quality for which his marginal benefit exceeds the marginal cost. So he will pay for additional quality up to the point where:

$$MB_A(Q) = MC(Q)$$

$$200 - Q = 5$$

$$Q = 195$$

Alex would by a couch with a quality of 195. At \$5 per unit of quality, this would cost Alex \$975.

(b) Suppose that Alex says he will pay the amount you find in part (a) for the shared couch. Assuming Alex keeps his promise, how much additional money will Bob be willing to pay for the shared couch?

Bob will pay for any additional units of quality beyond 195 for which his marginal benefit exceeds the cost of \$5. His current marginal benefit from an additional unit of couch quality, given that Alex has already paid for 195 units of quality, is:

$$MB_B(195) = 100 - \frac{1}{2} \cdot 195$$

$$MB_B(195) = 2.5$$

An additional unit of quality is only worth \$2.50 to Bob. Units beyond that are worth even less because Bob has diminishing marginal benefits from quality. Given that an additional unit of quality would cost \$5, Bob would have a negative net benefit from any additional couch quality. He will therefore choose to spend no additional money on the couch.

(c) Will the payments in part (b) lead to the efficient couch quality? Be certain to fully explain your answer including your calculation of the efficient couch quality.

The payments will be efficient if they lead to the level of couch quality for which total marginal benefit is equal to marginal cost. The current total marginal benefit is:

$$MB_{total}(195) = MB_A(195) + MB_B(195)$$
$$MB_{total}(195) = 5 + 2.5$$
$$MB_{total}(195) = 7.5$$

So at the current couch quality, the total marginal benefit of another unit of quality is \$7.50 which is more than an additional unit costs. Total surplus of the roommates would increase if they paid for more couch quality. The outcome of 195 units of quality is therefore not efficient. To find the efficient quality, we simply set total marginal benefit equal to marginal cost:

$$MB_{total}(Q) = MC(Q)$$
$$MB_A(Q) + MB_B(Q) = MC(Q)$$
$$200 - Q + 100 - \frac{1}{2}Q = 5$$
$$300 - \frac{3}{2}Q = 5$$
$$Q_{eff} \approx 196.7$$

As argued above, this efficient quality is higher then the couch quality the roommates end up with.

5. (20 points) Planting flowers in my garden is costly to me but I benefit from having a nicer looking garden. Every extra flower I plant increases my costs by \$10. This includes both the costs of buying the flowers and the costs of my time. For every extra flower I plant, the amount of enjoyment I get from my garden increases by an amount that depends on how many flowers, F, have already been planted. This marginal benefit from planting another flower is given by:

$$MB(F) = 100 - F \tag{7}$$

My neighbors do not incur any costs from me planting flowers but they do receive benefits from looking at a nicer garden. The benefits my neighbors get from an additional flower are half as big as the benefits I get. Draw a graph with the number of flowers on the horizontal axis that shows the items listed below. Label all intercepts, slopes and points of intersection with their numerical values:

- The marginal costs of planting an additional flower, $MC(F)$.
- My private marginal benefits from planting an additional flower, $MB_{private}(F)$.
- The social marginal benefits from planting an additional flower, $MB_{social}(F)$.
- The number of flowers I will choose to plant, F^*, if I am maximizing my individual surplus.
- The deadweight loss associated with my choice of F^*.
- The size of the subsidy per flower, S, that would lead me to choose the socially efficient number of flowers.

The graph below shows all of the relevant curves and points. The marginal cost curve is simply a horizontal line at \$10 given that each additional flower costs \$10 to plant. The private marginal benefit curve should have a vertical intercept of \$100 (found by plugging $F = 0$ into the private marginal benefit function) and a horizontal intercept of 100 flowers (found by setting $MB_{private}(F)$ equal to zero). The marginal benefits of the neighbors are one half of the private marginal benefits. Knowing this lets us find an equation for the neighbors' marginal benefit curve and for the social marginal benefit curve:

$$MB_{\text{neighbors}}(F) = \frac{1}{2} MB_{\text{private}}(F)$$

$$MB_{\text{neighbors}}(F) = 50 - \frac{1}{50} F$$

Note that this curve has the same horizontal intercept as the private marginal benefit curve but a vertical intercept of \$50 and a slope of $-\frac{1}{2}$ rather than -1. Now for the social marginal benefit curve:

$$MB_{\text{social}}(F) = MB_{\text{private}}(F) + MB_{\text{neighbors}}(F)$$

$$MB_{\text{social}}(F) = 100 - F + 50 - \frac{1}{2} F$$

$$MB_{\text{social}}(F) = 150 - \frac{3}{2} F$$

This is a downward sloping line with a vertical intercept of \$150, a horizontal intercept of 100 flowers and a slope of $-\frac{3}{2}$.

The number of flowers that I will plant will be where my marginal benefit curve intersects the marginal cost curve:

$$MB_{\text{private}}(F) = MC(F)$$

$$100 - F = 10$$

$$F^* = 90$$

So the equilibrium quantity of flowers planted, F^*, will be 90. The efficient number of flowers will be where the social marginal benefit curve intersects the marginal cost curve:

$$MB_{\text{social}}(F) = MC(F)$$

$$150 - \frac{3}{2}F = 10$$

$$F_{eff} = \frac{280}{3}$$

The deadweight loss at F^* will be equal to the area between the social marginal benefit curve and the marginal cost curve from F^* to F_{eff}. This area is given by:

$$DWL = \frac{1}{2}(MB_{\text{social}}(F^*) - MC(F^*))(F_{eff} - F^*)$$

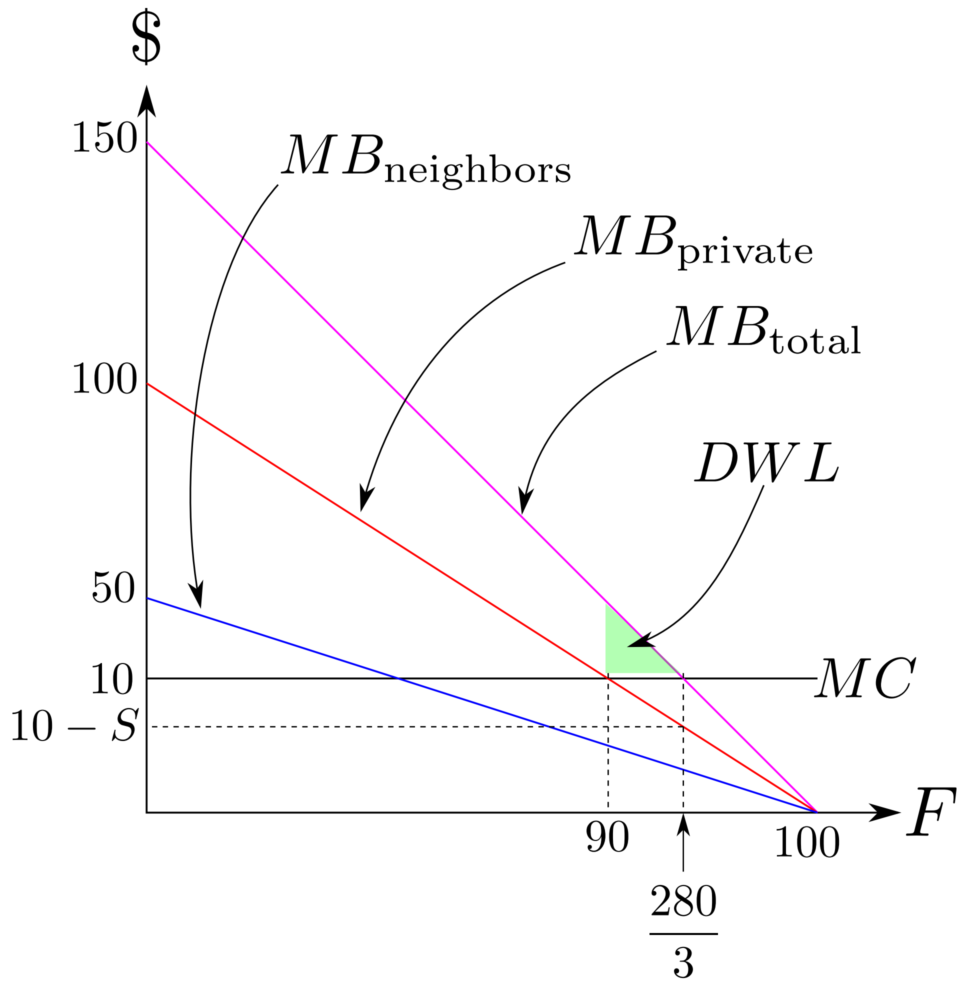
$$DWL = \frac{1}{2}(150 - \frac{3}{2} \cdot 90 - 10)(\frac{280}{3} - 90)$$

$$DWL \approx 8.3$$

To avoid this deadweight loss, we could provide a subsidy to induce me to plant the efficient number of flowers. The subsidy per unit should be equal to the size of the marginal externality at the efficient quantity. This is given by:

$$S = MB_{\text{neighbors}}(\frac{280}{3}) = 50 - \frac{1}{2} \cdot \frac{280}{3}$$

$$S = \frac{10}{3}$$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$MB_{\text{neighbors}}$
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