Ecn 100 - Intermediate Microeconomics
University of California - Davis October 20, 2010
Instructor: John Parman

Midterm 1 - Solutions

You have until 11:50am to complete this exam. Be certain to put your name, id number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the short answer questions directly on the exam. You must show your work for full credit. Answers may be left as fractions. Please place a box around final answers when appropriate. Good luck!

Name: **ID Number:** **Section:**

SECTION I: MULTIPLE CHOICE (60 points)

1. Suppose that the marginal utility from apples and the marginal utility from oranges are always positive. If there are no goods other than apples and oranges, a consumer who is maximizing utility will definitely:
   (a) Consume positive quantities of both apples and oranges.
   (b) Spend all of her income.
   (c) Both (a) and (b).
   (d) Neither (a) nor (b).

   (b) She can always increase utility by consuming more of one of the goods since marginal utility is positive. Therefore, if she is not spending all of her money, she could increase her utility by using the unspent income to buy more of one or both of the goods. We cannot say for certain that she will consume positive quantities of both goods. An example where she does not is the case of perfect substitutes. If the slope of the budget line differs from the constant slope of the indifference curves, she will consume all of one good and none of the other even though the marginal utility is positive for both goods.

2. If Alex is always willing to trade four books for one CD or one CD for four books:
   (a) The marginal utility of a CD is always smaller than the marginal utility of a book.
   (b) The marginal utility of a CD is always greater than the marginal utility of a book.
   (c) The marginal rate of substitution is increasing as the number of CDs increases.
   (d) The marginal rate of substitution is decreasing as the number of CDs increases.

   (b) Alex is always willing to trade between books and CDs at a constant rate. This tells us that the marginal rate of substitution is constant. If he is willing to trade four books for one CD, it must be the case that the marginal utility from one CD is four times as large as the marginal utility from one book.

3. Suppose that the demand equation for candy is given by $C = \frac{2I}{3p_C}$ where I is income and p_C is the price of a candy. The demand equation for soda is given by $S = \frac{I}{3p_S}$ where p_S is the price of soda. On a graph with candy on the vertical axis and soda on the horizontal axis, the price offer curve when varying the price of candy will be:

(a) A horizontal line.
(b) An upward sloping curve.
(c) A downward sloping curve.
(d) A vertical line.

(d) As the price of candy goes down, the quantity of candy consumed increases (since p_C is in the denominator of the demand equation for C). The price of candy does not show up in the demand equation for soda, so the change in the price of candy will have no effect on the quantity of soda consumed. This tells us that the price offer curve will trace out bundles with more and more candy but the same amount of soda as the price of candy drops. This is just a vertical line on a graph with candy on the vertical axis and soda on the horizontal axis.

4. Brian is indifferent between getting the bundle (10 sandwiches, 20 sodas) and the bundle (20 sandwiches, 10 sodas). If Brian's preferences are convex, we can say for certain that:

(a) Brian weakly prefers (15 sandwiches, 15 sodas) to (10 sandwiches, 20 sodas).
(b) Brian weakly prefers (20 sandwiches, 10 sodas) to (15 sandwiches, 15 sodas).
(c) Brian weakly prefers (10 sandwiches, 20 sodas) to (15 sandwiches, 15 sodas).
(d) Not enough information.

(a) If his preferences are convex and he is indifferent between (10 sandwiches, 20 sodas) and (20 sandwiches, 10 sodas), Brian will prefer an average of those bundles, (15 sandwiches, 15 sodas), to either of the extreme bundles.

5. On a graph with shoes on the horizontal axis and shirts on the vertical axis, an increase in the price of shoes would:

(a) Make the indifference curves steeper.
(b) Make the indifference curves flatter.
(c) Make the budget line steeper.
(d) Make the budget line flatter.

(c) Notice that magnitude of the slope of the budget line is equal to the price of shoes divided by the price of shirts. If the price of shoes goes up, the magnitude of the slope gets larger meaning a steeper budget line. The shapes of the indifference curves do not depend on the prices, only on the utility function.

6. Which of the following pairs of utility functions represent the same preferences?

(a) $U(x,y) = 2x + 2y$, $U(x,y) = -2x - 2y$.
(b) $U(x,y) = 2xy$, $U(x,y) = x^2y^2$.
(c) $U(x,y) = 2xy$, $U(x,y) = 2x + 2y$.
(d) $U(x,y) = 2x + 2y$, $U(x,y) = 2x^2 + 2y^2$.

(b) The only two pairs of utility functions that give the same marginal rate of substitution for each function are (a) and (b). That tells us that these are the only two cases where the indifference curves have the same shape for both functions. For (a), x and y are goods in the first function and bads in the second function, so the two functions actually represent completely opposite preferences. For (b), x and y

are good in both cases, so not only do the indifference curves have the same shape, utility is increasing in the same direction for both functions.

7. Suppose that tacos are an ordinary good and that tacos and salsa are complements. If the price of tacos goes down:

   (a) Spending on salsa will go up.
   (b) Spending on salsa will go down.
   (c) Consumption of tacos will go down.
   (d) Not enough information.

   (a) If the price of tacos goes down, consumption of tacos will go up because they are ordinary and consumption of salsa will go up since it is a complement. Because the price of salsa has not changed, an increase in salsa consumption will lead to an increase in spending on salsa. It is unclear how spending on tacos changes (the increase in quantity and decrease in price have opposite effects on total spending).

8. Christine is currently maximizing her utility by consuming a bundle with two books and two magazines. If her preferences are monotonic, then a bundle with two books and three magazines:

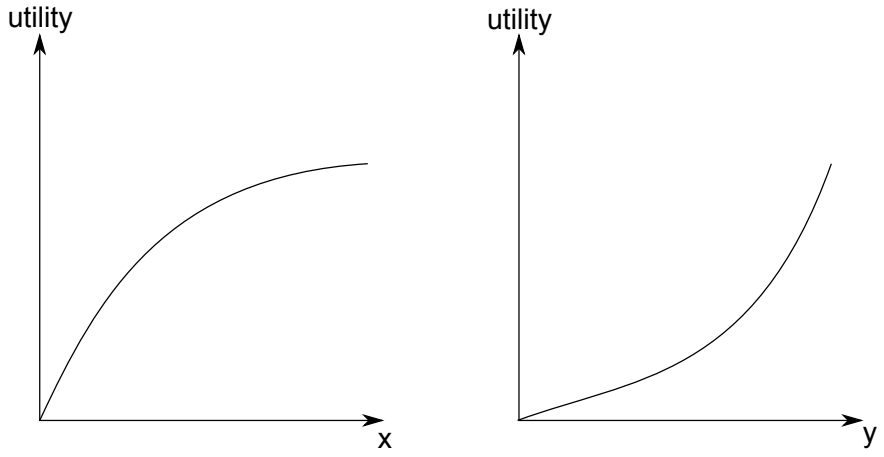
   (a) Is not in Christine's budget set.
   (b) Would give Christine a higher level of utility.
   (c) Both (a) and (b).
   (d) Neither (a) nor (b).

   (c) If preferences are monotonic, we get downward sloping indifference curves. This means that a bundle with the same amount of one good and more of the other good will lie above the current indifference curve giving Christine higher utility. If she was maximizing utility and did not consume this bundle that gives higher utility, it must be the case that the bundle is not affordable.

9. Doug is indifferent between all bundles on his budget line and prefers any of these bundles to any bundle below his budget line. Doug is currently maximizing his utility. If the price of good x is \$4 and the price of good y is \$8, then on a graph with x on the horizontal axis and y on the vertical axis:

   (a) Doug's indifference curve at his current bundle has a slope of -2.
   (b) Doug's indifference curve at his current bundle has a slope of $-\frac{1}{2}$.
   (c) Doug's marginal rate of substitution is diminishing when moving from left to right along his current indifference curve.
   (d) None of the above.

   (b) If Doug is indifferent between all bundles on the budget line, his current indifference curve must be a straight line with a slope equal to that of the budget line. The budget line slope is simply $-\frac{p_x}{p_y}$ or $-\frac{4}{8}$. Since the indifference curve is a straight line, Doug's marginal rate of substitution is constant, not diminishing.



Use the figure above to answer questions 10 through 12. The graph on the left shows the relationship between x and utility holding y constant. The graph on the right shows the relationship between y and utility holding x constant.

10. Based on the two graphs, which of the following statements are true?
    (a) The marginal utility of x is increasing.
    (b) The marginal utility of y is increasing.
    (c) Both (a) and (b).
    (d) Neither (a) nor (b).

    (b) On a graph of utility as a function of an individual good, the slope of the curve is equal to the marginal utility of that good. On the graph for x, the slope gets smaller as x gets bigger meaning that the marginal utility for x is decreasing. For y, the slope gets steeper as y gets larger implying an increasing marginal utility.

11. On a graph with x on the horizontal axis and y on the vertical axis, which of the following is definitely true?
    (a) The indifference curves would have a negative slope.
    (b) The indifference curves would have a positive slope.
    (c) The indifference curves would get steeper when moving from left to right.
    (d) The indifference curves would get flatter when moving from left to right.

    (a) Utility increases when x increases and when y increases, telling us that both x and y are goods. Indifference curves for two goods have a downward slope. We cannot say whether the indifference curves are getting flatter or steeper as we move from left to right. When moving from left to right along an indifference curve, x is getting larger and y is getting smaller. Since the marginal utility of x is decreasing as x gets larger and the marginal utility of y is decreasing as y gets smaller, both the numerator and denominator in our expression for the MRS ($-\frac{MU_x}{MU_y}$) are getting smaller. If MU_y is getting smaller at a faster rate, the MRS will be increasing. If MU_x gets smaller at a faster rate the MRS will be decreasing.

12. Which of the following is true?

(a) Both x and y are goods.
(b) Both x and y are bads.
(c) x is a good and y is a bad.
(d) x is a bad and y is a good.

(a) For both x and y, increasing the quantity of the good increases utility. This is the definition of a good.

13. Suppose that the price of carrots doubles and the price of lettuce triples. If income also triples:

(a) The set of affordable bundles will be larger than before any of the price and income changes.
(b) The set of affordable bundles will be smaller than before any of the price and income changes.
(c) There will be no change in the set of affordable bundles.
(d) Some bundles will now be affordable that weren't affordable before and some bundles that were affordable before will no longer be affordable.

(a) Think about the end points of the budget line. Originally, these endpoints would be $\frac{I}{p_c}$ on the carrots axis and $\frac{I}{p_l}$ on the lettuce axis. After the price and income changes, the new endpoints would be $\frac{3I}{2p_c}$ on the carrots axis and $\frac{3I}{3p_l}$ on the lettuce axis. Notice that the new carrot endpoint is 1.5 times bigger than the original endpoint and the new lettuce endpoint is exactly the same as the original endpoint. This tells us that the net effect of the price and income changes was an outward rotation of the budget line, increasing the set of affordable bundles.

14. If comic books are an inferior good, the Engel curve for comic books will be:

(a) Upward sloping.
(b) Downward sloping.
(c) A horizontal line.
(d) A vertical line.

(b) If comic books are an inferior good, the quantity of comic books will increase as income decreases. This will produce a downward sloping line on graph with income on the vertical axis and comic books on the horizontal axis.

15. If Ellen considers tuna a good and sardines a bad and those are the only two things she can consume, Ellen will:

(a) Spend all of her money on tuna.
(b) Spend all of her money on sardines.
(c) Spend money on both tuna and sardines but more on tuna than sardines.
(d) Spend money on both tuna and sardines but more on sardines than tuna.

(a) More money spent on tuna will always increase utility while more money spent on sardines will always decrease utility. In order to maximize utility, all money should be spent on tuna.

SECTION II: SHORT ANSWER (40 points)

1. (15 points) You have ten hours to study for a chemistry exam and a biology exam. If you spend B hours studying for biology and C hours studying for chemistry, your utility is given by:

$$U(B,C) = 10BC \tag{1}$$

   (a) Write down an equation for your budget constraint in terms of B and C. Note that your budget constraint will be in hours, not dollars.

   Our constraint is that we only have ten hours to study, so the sum of the hours spent on biology and the hours spend on chemistry has to be less than or equal to ten:

$$B + C \leq 10$$

   It is fine if you wrote the constraint using an equal sign rather than a less than or equal to symbol.

   (b) Derive an expression for the marginal rate of substitution. Your marginal rate of substitution should be expressed as the change in biology study hours relative to a change in chemistry study hours. In other words, if you were willing to give up one hour of studying biology to get two more hours of study time for chemistry, the marginal rate of substitution would be $-\frac{1}{2}$.

   To get the marginal rate of substitution we first need to derive the marginal utilities:

$$MU_B = \frac{dU(B,C)}{dB} = 10C$$

$$MU_C = \frac{dU(B,C)}{dC} = 10B$$

   We want the marginal rate of substitution in the form of $\frac{\Delta B}{\Delta C}$. Recall that in class we showed that the MRS expressed as $\frac{\Delta y}{\Delta x}$ was equivalent to $-\frac{MU_x}{MU_y}$. So in this case, the marginal rate of substitution is:

$$MRS = \frac{\Delta B}{\Delta C} = -\frac{MU_C}{MU_B} = -\frac{10B}{10C} = -\frac{B}{C}$$

   (c) Given your budget constraint from part (a) and your utility function, solve for the optimal way to divide your studying time. Be certain to show all of your work.

   First we need to set up our tangency condition:

$$-\frac{p_C}{p_B} = MRS$$

   Note that our budget constraint is in terms of hours, so our prices will also be in terms of hours. It costs one hour to get one additional unit of B or one additional unit of C. So p_B and p_C are both equal to one. Plugging in these prices and our expression for the MRS gives us:

$$-\frac{1}{1} = -\frac{B}{C}$$

Now we can rearrange to get C by itself:

$$B = C$$

Plugging this result into the budget constraint gives us:

$$p_B B + p_C C = 10$$

$$1 \cdot B + 1 \cdot B = 10$$

$$2B = 10$$

$$B = 5$$

So the optimal amount of study time for biology is 5 hours. Since the tangency condition showed us that B is equal to C, the optimal study time for chemistry is also 5 hours.

2. (10 points) The demand equation for video games is given by:

$$V = \frac{I}{p_V + \frac{p_V^2}{p_C}} \tag{2}$$

where V is the optimal number of video games, p_V is the price of a video game, p_C is the price of a CD, and I is income. The demand equation for CDs is given by:

$$C = \frac{I}{p_C + \frac{p_C^2}{p_V}} \tag{3}$$

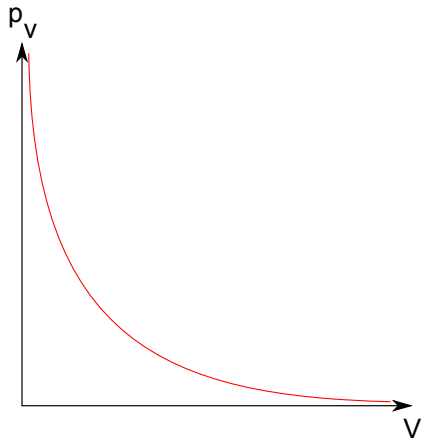
where C is the optimal number of CDs. Suppose that the current price of a CD is \$10, the price of video game is \$20 and your current income is \$100.

(a) Draw the demand curve for video games. Label any intercepts and slopes with their numerical values whenever possible. Also be certain to clearly label the axes.

The demand curve shows the price of video games on one axis and the quantity demanded on the other axis. This means that we are holding income and the price of CDs constant and can simply plug in their current values into the demand function giving us:

$$V = \frac{100}{p_V + \frac{p_V^2}{10}}$$

Notice that as p_V gets close to zero, V is approaching infinity. As p_V gets very large, V approaches zero. This leads to a downward sloping demand curve that approaches the vertical axis as V approaches zero and approaches the horizontal axis as V approaches infinity.



(b) Draw the Engel curve for CDs. Label any intercepts and slopes with their numerical values whenever possible. Also be certain to clearly label the axes.

For the Engel curve, prices are being held constant and income is allowed to vary. Plugging in our prices leads to the following simplified demand equation

for CDs:

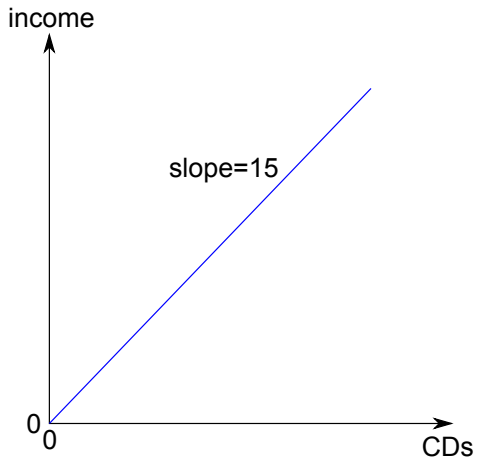
$$C = \frac{I}{10 + \frac{10^2}{20}}$$

$$C = \frac{I}{15}$$

This demand equation gives C as a function of I. When graphing the Engel curve, the curve corresponds to I as a function of C. So it is useful to rearrange the demand equation to get I by itself:

$$I = 15C$$

This is the equation of a straight line with a slope of 15 that passes through the origin.



3. (15 points) Gary always makes burgers by using one bun for every two hamburger patties. Every completed burger (one bun, two patties) gives Gary one additional unit of utility. Gary only gets utility from buns and patties by consuming them together. Any leftover buns or patties do not give Gary any additional utility. For example, if he has two buns and five patties, he would make two completed burgers giving him a total of two units of utility. The fifth patty would have no effect on his utility.

   (a) Write down a utility function that represents the utility Gary gets from B buns and H hamburger patties.

   From the description of this preferences, Gary considers buns and patties to be perfect complements; he always consumes them in a fixed proportion. This tells us that we need to use a min function:

   $$U(B,H) = min(\alpha B, \beta H)$$

   We need to figure out what values to choose for the coefficients α and β. First, note that Gary always consumes one bun for every two patties. This tells us that what will limit Gary's utility is whichever is smaller, the number of buns or one half the number of patties. So β should be half as big as α. Second, we know that gary gets one unit of utility from one completed burger, so plugging in one for B and two for H should give us a utility of one. The only pair of coefficients that satisfies both of these criteria is α equal to one, β equal to one half:

   $$U(B,H) = min(B, \frac{1}{2}H)$$

   Let's double check this utility function using the two buns, five patties example given in the description:

   $$U(2,5) = min(2, \frac{1}{2} \cdot 5)$$

   $$U(2,5) = min(2, 2.5)$$

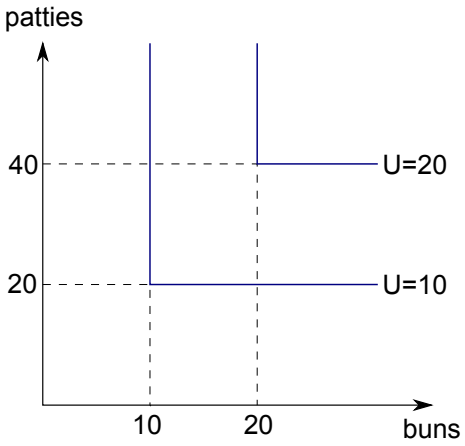
   $$U(2,5) = 2$$

   Our function gives us the correct answer.

   (b) Gary currently has four buns and nine patties. What is the marginal utility of patties at this particular bundle?

   The marginal utility of patties at this current bundle is zero. We cannot calculate this by taking derivatives. Instead, we need to just apply a little bit of logic. The marginal utility of patties is the increase in utility from a one unit increase in patties. Gary already has too many patties. With four buns, he can only use eight of the nine patties. The ninth patty has no effect on his utility. Similarly, adding a tenth patty would have no effect on his utility. So the change in utility from a one unit increase in patties would be zero.

   (c) On a graph with with buns on the horizontal axis and patties on the vertical axis, sketch the indifference curves for a utility level of 10 and a utility level of 20. Label any relevant intercepts, kinks and slopes with their numerical values.


patties
40
20
U=20
U=10
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buns