Midterm 1 - Solutions

You have until 11:50am to complete this exam. Be certain to put your name, id number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the long answer questions directly on the exam. You must show your work for full credit. Answers may be left as fractions. Please place a box around final answers when appropriate. Good luck!

Name: **ID Number:**   **Section:**

SECTION I: MULTIPLE CHOICE (60 points)

1. Which of the following pairs of utility functions would represent the same preferences?

   (a) $U(x, y) = 2xy$ and $V(x, y) = xy$.
   (b) $U(x, y) = x^2 y^2$ and $V(x, y) = xy$.
   (c) Neither (a) nor (b).
   (d) Both (a) and (b).

   > (d) Notice that in both cases, the two utility functions would give you exactly the same marginal rate of substitution ($MRS = -\frac{y}{x}$) and that x and y are both goods. This tells us that both utility functions would generate the same indifference curves and represent the same preferences.

2. In which of the following cases would the demand curve for good x and the Engel curve for good x have the same sign?

   (a) When good x is a normal, ordinary good.
   (b) When good x is an inferior, Giffen good.
   (c) When good x is an inferior, ordinary good.
   (d) Both (b) and (c).

   > (c) If x is an inferior good the Engel curve will be downward sloping (when income goes up, demand goes down). If x is an ordinary good, the demand curve will be downward sloping (when price goes up, demand goes down).

3. Suppose that every extra cookie increases Abe's utility by four units. Which of the following must be true?

   (a) Abe's marginal utility of cookies is zero.
   (b) Abe's marginal utility of cookies is increasing.
   (c) Abe's marginal utility of cookies is diminishing.
   (d) Abe's marginal utility of cookies is positive.

   > (d) If an extra cookie increases utility, marginal utility must be positive. Since every extra cookie increases utility by four units, the marginal utility of cookies is constant (and equal to four).

4. Betty is indifferent between eating 2 apples and 6 oranges and eating 6 apples and 2 oranges. Betty prefers eating 4 apples and 4 oranges to eating 2 apples and 6 oranges. We can say for certain that: **(Note: This was changed at the beginning of the exam to say Betty prefers (2 apples, 6 oranges) to (4 apples, 4 oranges).)**

   (a) Betty's preferences are not transitive.
   (b) Betty's preferences are not convex.
   (c) Betty's preferences are not monotonic.
   (d) (a) and (b).

   (b) Notice that the bundle $(4, 4)$ is an average of the other two bundles she is indifferent between, $(2, 6)$ and $(6, 2)$. The information in the problem reveals that Betty prefers either of these two extreme bundles to the average bundle. Therefore her preferences cannot be convex.

5. At his current bundle, Calvin's marginal utility from pretzels is 2 and his marginal utility from cans of soda is 4. What is the largest number of pretzels Calvin would be willing to trade for one more can of soda?

   (a) 1.
   (b) 2.
   (c) 4.
   (d) 8.

   (b) A can of soda gives Calvin twice as much utility as a pretzel. So Calvin would be willing to give up two pretzels to get one can of soda. If he gave up more than two, his utility would go down as a result of the trade.

6. Suppose that coffee and sugar are complements. The slope of the price offer curve when the price of coffee is varied will be: **(Note: Question was modified in the exam to say that coffee is an ordinary good.)**

   (a) Positive.
   (b) Negative.
   (c) Zero.
   (d) It depends on which good is on the horizontal axis.

   (a) As the price of coffee goes up, both the amount of coffee and the amount of sugar consumed will decrease since coffee is ordinary and sugar is a complement.

7. Dotty is currently spending all of her money on shirts and jackets and these are the only two goods she consumes. At her current bundle, a dollar spent on shirts would increase utility more than a dollar spent on jackets. At this current bundle on a graph with shirts on the horizontal axis:

   (a) The indifference curve is steeper than the budget line.
   (b) The indifference curve is flatter than the budget line.
   (c) The indifference curve is tangent to the budget line.
   (d) There is no indifference curve passing through the bundle.

   (a) The information from the problem tells us that the marginal utility of shirts
   relative to the marginal utility of jackets is greater than the price of shirts relative
   to the price of jackets. So the slope of the indifference curve is larger in magnitude
   than the slope of the budget line.

8. On a graph with pencils on the horizontal axis and erasers on the vertical axis, the greater
   the price of pencils is:

   (a) The steeper the indifference curves will be.
   (b) The flatter the indifference curves will be.
   (c) The steeper the budget line will be.
   (d) The flatter the budget line will be.

   (c) The slope of the budget line is determined by the relative prices, specifically the
   price of pencils divided by the price of erasers. The more expensive pencils get, the
   steeper the slope of the budget line wil be.

9. Eugene likes salt and pepper and always consumes them in equal quantities. An increase in
   the price of pepper will lead to (you can assume Eugene spends all of his income on salt and
   pepper):

   (a) An increase in the quantity of pepper consumed and a decrease in the quantity of salt
       consumed.
   (b) An increase in the quantity of salt consumed and a decrease in the quantity of pepper
       consumed.
   (c) An increase in both the quantity of salt consumed and the quantity of pepper consumed.
   (d) A decrease in both the quantity of salt consumed and the quantity of pepper consumed.

   (d) With perfect complements consumed in a one to one ratio, the demand for each
   good will be equal to income divided by the sum of the two prices. If either price
   goes up, demand for both goods will go down.

10. Suppose that when graphing utility as a function of good x, with good x on the horizontal
    axis, we get a downward sloping curve. Then we know that:

    (a) The marginal utility of x is diminishing.
    (b) The marginal utility of x is positive.
    (c) x is a bad.
    (d) The marginal utility of x is constant.

    (c) A downward sloping utility curve tells us that as x increases, utility goes down.
    This means x is a bad (it also tells us that the marginal utility of x is negative).

11. The demand for jelly beans (J) in terms of the price of jelly beans (p_J), the price of chocolates
    (p_C) and income (I) is given by the following demand function:

    $$J(p_J, p_C, I) = \frac{I}{2p_J + 5p_C}$$

    Which of the following is true?

    (a) Jelly beans are an ordinary good, jelly beans and chocolates are substitutes.

(b) Jelly beans are a normal good, jelly beans and chocolates are complements.

(c) Jelly beans are an inferior good, jelly beans and chocolates are substitutes.

(d) Jelly beans are a Giffen good, jelly beans and chocolates are complements.

(b) From the demand function, we can see that if income increases, demand for jelly beans goes up so jelly beans are a normal good. We can also see that if the price of chocolates increases demand for jelly beans goes down so jelly beans and chocolates are complements.

12. If two bundles lie on the same indifference curve, which of the following is definitely true?

(a) Both bundle cost the same amount.

(b) The bundles may lie on different indifference curves if prices change.

(c) The more expensive of the two bundles will give the consumer greater utility.

(d) The utility level of each bundle is the same.

(d) By definition, two bundles on the same indifference curve give the consumer the same level of utility. Prices will not affect whether the bundles are on the same indifference curve.

13. If good x and good y are the only two goods a person consumes, doubling the prices of both goods and doubling income will:

(a) Make the set of affordable bundles larger.

(b) Make the set of affordable bundles smaller.

(c) Leave the set of affordable bundles unchanged.

(d) Change the consumer's optimal bundle.

(c) Notice that the slope of the budget line stays the same ($\frac{p_x}{p_y} = \frac{2p_x}{2p_y}$) and the end points of the budget stay the same ($\frac{I}{p_x} = \frac{2I}{2p_x}$). So the budget line and the set of affordable bundles stay the same.

14. Suppose Francine considers both spiders and roaches to be bads. On a graph with spiders on the horizontal axis and roaches on the vertical axis:

(a) Francine's indifference curves will have a positive slope.

(b) Francine's indifference curves will have a negative slope.

(c) Francine's utility will be increasing as the number of spiders and roaches increases.

(d) Francine's utility will be decreasing as the number of spiders and roaches decreases.

(b) The indifference curves will be downward sloping. Adding additional spiders would decrease utility, so to keep Francine indifferent you would have to take away roaches.

15. Suppose that at George's optimal bundle, he buys positive quantities of both books and DVDs, the slope of the indifference curve is equal to -1 and he spends all of his income on books and DVDs. Which of the following must be true?

(a) The price of a book is equal to the price of a CD.

(b) The quantity of books he buys is equal to the quantity of CDs he buys.

(c) Both (a) and (b).

(d) Neither (a) nor (b).

(a) If George is at his optimal bundle, the indifference curve will be tangent to the budget line. The slope of the indifference curve at his optimal bundle is -1 so this will also be the slope of the budget line. A budget line with a slope of -1 tells us that the prices of the two goods are equal.

SECTION II: SHORT ANSWER (40 points)

1. (16 points) Harriet's utility from board games (B) and toys (T) is given by the following utility function:
$$U(B,T) = B^{\frac{1}{2}} + 4T^{\frac{1}{2}}$$

(a) Derive expressions for the marginal utility of board games and for the marginal utility of toys.

$$MU_B = \frac{dU(B,T)}{dB} = \frac{1}{2}B^{-\frac{1}{2}} + 0 = \frac{1}{2}B^{-\frac{1}{2}}$$

$$MU_T = \frac{dU(B,T)}{dT} = 0 + 4 \cdot \frac{1}{2}T^{-\frac{1}{2}} = 2T^{-\frac{1}{2}}$$

(b) Derive an expression for the optimal number of board games in terms of the price of board games (p_B), the price of toys (p_T) and Harriet's income (I).

We begin by setting up our tangency condition, setting the slope of the budget line equal to the slope of the indifference curve, and solving for one of the two goods:

$$\frac{p_B}{p_T} = \frac{MU_B}{MU_T}$$

$$\frac{p_B}{p_T} = \frac{\frac{1}{2}B^{-\frac{1}{2}}}{2T^{-\frac{1}{2}}}$$

$$\frac{p_B}{p_T} = \frac{T^{\frac{1}{2}}}{4B^{\frac{1}{2}}}$$

$$\frac{p_B^2}{p_T^2} = \frac{T}{16B}$$

$$16\frac{p_B^2}{p_T^2}B = T$$

Now we can plug this expression for T into the budget constraint:

$$p_B B + p_T T = I$$

$$p_B B + p_T 16\frac{p_B^2}{p_T^2}B = I$$

$$B\left(p_B + 16\frac{p_B^2}{p_T}\right) = I$$

$$B = \frac{I}{p_B + 16\frac{p_B^2}{p_T}}$$

This is our demand equation giving us the optimal number of board games in terms of prices and income.

2. (12 points) Isaac buys music albums both in the form of CDs and as digital downloads. His utility from albums on CD (C) and digital albums (D) is given by:

$$U(C, D) = 8C + 10D$$

(a) Suppose that Isaac is offered a choice between one free CD or one free digital album. Which will he take? Explain your answer. (Your explanation should only require one or two sentences.)

> The marginal utility from a CD (8) is less than the marginal utility of a digital album (10). So if they both cost the same amount, in this case a price of zero, Isaac will choose the digital album since it offers higher utility.

(b) The price of a CD is \$5 and the price of a digital album is \$6 and Isaac's income is \$30. Draw two graphs, one showing the Engel curve for CDs and one showing the Engel curve for digital albums. Be certain to clearly label slopes and intercepts with their values and to label the axes.

> Notice that the marginal utility of a CD relative to its price if $\frac{8}{5}$ or 1.6 while the marginal utility of a digital album relative to its price is $\frac{10}{6}$ or 1.67. So a dollar spent on digital albums generates more utility than a dollar spent on CDs. Isaac will spend all of his money on digital albums and none of it on CDs. This gives us the following two demand functions:

$$D = \frac{I}{p_D} = \frac{I}{6}$$

$$C = 0$$

> These two equations give us all of the information we need to graph the Engel curves.
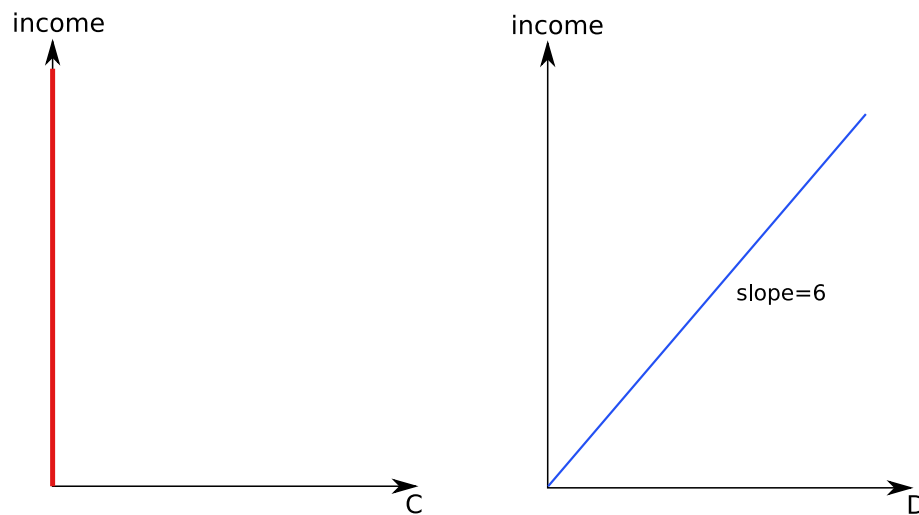
3. (12 points) For each scenario below, draw the graph that the question asks for, labeling as much as you can.

   (a) Suppose that apples and bananas are the only two goods a person buys. Initially the price of an apple is $2, the price of a banana is $4 and the person's income is I. Then the price of apples goes up to $4 and the person's income doubles. On a graph with apples on the horizontal axis and bananas on the vertical axis, show both the original budget line and the new budget line.

   The slope of the budget line will be $-\frac{p_a}{p_b}$. So initially, the slope is $-\frac{2}{4}$. After the price change, the slope will be $-\frac{4}{4}$. So the new budget line is twice as steep as the old budget line. In terms of the endpoints, the intercept on the apple axis will be $\frac{I}{p_a}$. Initially this is $\frac{I}{2}$. After the price and income change, this is $\frac{2I}{4}$ which reduces to $\frac{I}{2}$. So the apple intercept hasn't changed. For the intercept on the banana axis, the intercept is $\frac{I}{p_b}$. Initially this is $\frac{I}{4}$. After the price and income change, this increases to $\frac{2I}{4}$ or $\frac{I}{2}$.
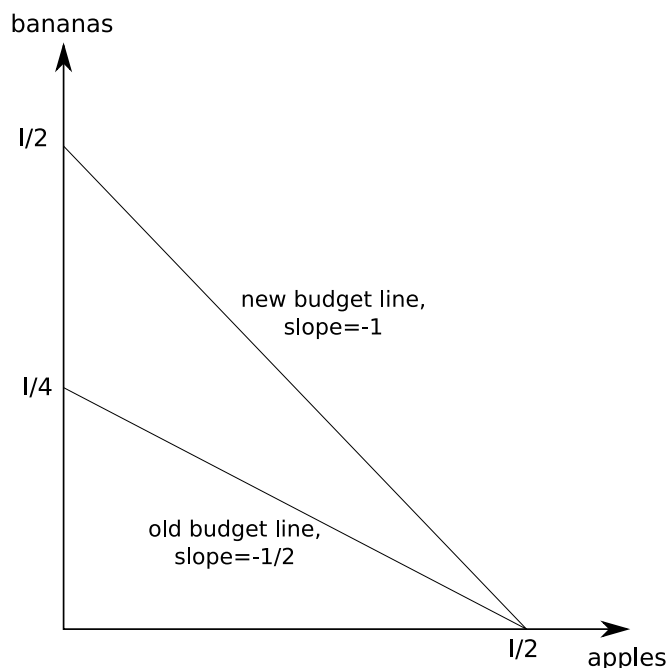
   (b) Suppose apples and bananas are both normal, ordinary goods and are substitutes. On a graph with apples on the horizontal axis and bananas on the vertical axis, show three points on the price offer curve obtained by changing the price of apples. Be certain to show the budget line and indifference curve corresponding to each point.

   As the price of apples goes down, the quantity of apples in the optimal bundle should increase because apples are ordinary and the quantity of bananas in the optimal bundle should decrease because apples and bananas are substitutes. So our price offer curve will be downward sloping: