Ecn 102 - Analysis of Economic Data
University of California - Davis May 19, 2009
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Midterm 2 - Solutions

You have until 10:20am to complete this exam. Please remember to put your name, section and ID number on both your scantron sheet and the exam. Fill in test form A on the scantron sheet. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each multiple choice question. Answer the long answer questions directly on the exam. Keep your answers complete but concise. For the long answer questions, you must show your work for full credit.

Name: **ID Number:** **Section:**

(POTENTIALLY) USEFUL FORMULAS AND EXCEL OUTPUT

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$CV = \frac{s}{\bar{x}}$

$skew = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}(\frac{x_i-\bar{x}}{s})^3$

$kurt = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}(\frac{x_i-\bar{x}}{s})^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$

$\mu = E(X)$

$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

$Pr[T_{n-k} > t_{\alpha,n-k}] = \alpha$

$Pr[|T_{n-k}| > t_{\frac{\alpha}{2},n-k}] = \alpha$

$\sum_{i=1}^{n} a = na$

$\sum_{i=1}^{n}(ax_i) = a\sum_{i=1}^{n} x_i$

$\sum_{i=1}^{n}(x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

$s^2 = \bar{x}(1-\bar{x})$ for proportions data

$t_{\alpha,n-k} = TINV(2\alpha, n-k)$

$Pr(|T_{n-k}| \geq |t^*|) = TDIST(|t^*|, n-k, 2)$

$Pr(T_{n-k} \geq t^*) = TDIST(t^*, n-k, 1)$

$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

$r_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \cdot \sum_{i=1}^{n}(y_i-\bar{y})^2}}$

$r_{xy} = \frac{s_{xy}}{\sqrt{s_{xx} \cdot s_{yy}}}$

$\hat{y}_i = b_1 + b_2 x$

$e_i = y_i - \hat{y}_i$

$b_2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

$b_1 = \bar{y} - b_2\bar{x}$

$b_2 = r_{xy}\sqrt{\frac{s_{yy}}{s_{xx}}}$

$TSS = \sum_{i=1}^{n}(y_i - \bar{y})^2$

$ESS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$R^2 = 1 - \frac{ESS}{TSS}$

$E(b_2) = \beta_2$

$s_{b_2} = \sqrt{\frac{s_e^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$

$s_e^2 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$t^* = \frac{b_2 - \beta_2^*}{s_{b_2}}$

TINV(.005,50)=2.94 TINV(.02,50)=2.40 TINV(.01,49)=2.68
TINV(.01,50)=2.68 TINV(.005,49)=2.94 TINV(.02,49)=2.40

SECTION I: MULTIPLE CHOICE (60 points)

1. Which of the following statements is always true?
   (a) The correlation between two variables is greater than or equal to the covariance.
   (b) The covariance between two variables is greater than or equal to the correlation.
   (c) The correlation between x and y will have the same sign as the slope coefficient from a regression of y on x.
   (d) The correlation between x and y will have the opposite sign of the slope coefficient from a regression of y on x.

   (c) The slope coefficient is equal to the correlation times the square root of the ratio of the variances of x and y. Since the variances of x and y will always be positive, the sign of the correlation will determine the sign of the slope coefficient.

2. Which of the following would reduce the standard error of the slope coefficient?
   (a) Decreasing the number of observations.
   (b) Greater variation in the independent variable.
   (c) Having data points that are further from the regression line.
   (d) None of the above.

   (b) Greater variation in the independent variable will increase $\sum_{i=1}^{n}(x_i - \bar{x})^2$ in the denominator of the standard error equation, reducing the size of the standard error.

3. Suppose that you run a regression of y on x. Your 95% confidence interval for the slope cofficient turns out to be (2.8, 3.2). Which of the following statements are definitely true?
   (a) The slope coefficient is statistically significant at the 5% significance level.
   (b) The slope coefficient is economically significant at the 5% significance level.
   (c) The slope coefficient is economically significant at the 10% significance level.
   (d) (a) and (b).

   (a) The 95% confidence interval does not contain zero. This tells us that we can reject that the coefficient is zero at the 5% significance level. However, this is only telling us about statistical significance. To make conclusions on the economic significance of the variable we would need to know whether the magnitude of the coefficient is large enough to be meaningful.

4. Which of the following statements is true?
   (a) The regression line will always pass through at least one data point.
   (b) The regression line will pass through the point $(\bar{x}, \bar{y})$.
   (c) The regression line will always pass through the origin.
   (d) (a) and (b).

   (b) We solve for the value of b_1 by using the equation $b_1 = \bar{y} - b_2\bar{x}$. So by construction the regression line will pass through $(\bar{x}, \bar{y})$.

5. Suppose there is an exponential growth relationship between x and y (y is growing exponentially as x increases). which of the following regression equations would you use to estimate the relationship between x and y?

(a) $y = b_1 + b_2 x$.
(b) $y = b_1 + b_2 \ln x$.
(c) $\ln y = b_1 + b_2 x$.
(d) $\ln y = b_1 + b_2 \ln x$.

(c) A log-linear model is used when y is growing exponentially. In this case, the percentage change in y will be proportional to a one unit change in x.

6. If two variables are perfectly correlated, when we regress one variable on the other, which of the following will definitely be true?
   (a) The estimated intercept will be equal to zero.
   (b) The estimated slope coefficient will be equal to one.
   (c) The R^2 will be equal to one.
   (d) The covariance will be equal to one.

   (c) If two variables are perfectly correlated, variation in one variable will perfectly predict any variation in the other. In this case, the error sum of squares would be zero making the R^2 equal to one.

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	11.7	0.35	33.07	5.23E-83	10.98	12.37
HUMIDITY	-0.05	0.0063	-7.22	1.04E-11	-0.058	-0.033

Use the regression output above to answer questions (7) through (10). The output corresponds to a regression of visibility (measured in miles) on relative humidity (measured as a percentage). The observations are daily observations over a seven month period. A day with 10 miles of visibility and a relative humidity of 30% would be entered as $(10, 30)$ in the dataset.

7. Which of the following conclusions would you make based on the regression results?
   (a) There is a positive, statistically significant (at a 5% significance level) relationship between humidity and visibility.
   (b) There is a positive but statistically insignificant (at a 5% significance level) relationship between humidity and visibility.
   (c) There is a negative, statistically significant (at a 5% significance level) relationship between humidity and visibility.
   (d) There is a negative but statistically insignificant (at a 5% significance level) relationship between humidity and visibility.

   (c) The coefficient on humidity is negative indicating a negative relationship between humidity and visibility. The p-value for the coefficient is less than .05, meaning that the coefficient is statistically significant at the 5% significance level.

8. What is the predicted visibility when relative humidity is 50%?
   (a) 585 miles.

(b) 9.2 miles.
(c) 5.8 miles.
(d) 11.7 miles.

(b) The predicted visibility will be $11.7 - .05 \cdot 50$ which is 9.2 miles.

9. Suppose b_2^{upper} is the upper end of the 90% confidence interval for the slope coefficient. Which of the following is definitely true?

(a) $b_2^{upper} > -.033$.
(b) $b_2^{upper} < -.033$.
(c) $b_2^{upper} > 12.37$.
(d) Not enough information.

(b) Note that the upper bound of the 95% confidence interval is -.033. The 90% confidence interval will be narrower than the 95% confidence interval but still centered at the same value, so the upper bound of the 90% confidence interval will be less than -.033.

10. If the average humidity in the sample was 80%, what was the average visibility?

(a) 11.7 miles.
(b) 12.7 miles.
(c) 15.7 miles.
(d) 7.7 miles.

(d) We know that $\bar{y} = b_1 + b_2\bar{x}$. So average visibility in the sample will be $11.7 - .05 \cdot 80$ or 7.7 miles.

11. Suppose that we run a regression of heart rate on days of exercise per week and get a negative t-stat for the slope coefficient. If the magnitude of the t-stat is large enough to reject the null hypothesis that the coefficient is zero at a 5% significance level, which of the following statements may be false?

(a) We would reject the null hypothesis that the coefficient is greater than or equal to zero at a 5% significance level.
(b) We would reject the null hypothesis that the coefficient is greater than or equal to zero at a 2.5% significance level.
(c) We would reject the null hypothesis that the coefficient is less than or equal to zero at a 5% significance level.
(d) Both (b) and (c) may be false.

(c) We will not reject the null hypothesis that the coefficient is less than or equal to zero at any significance level because we ended up with a negative t-stat.

12. Which of the following would not lead to a wider confidence interval for the slope coefficient when regressing y on x?

(a) Making the significance level (α) larger.
(b) A larger standard error for b_2.
(c) A smaller sample variance for x.
(d) A smaller sample size.

(a) A larger α leads to narrower confidence interval (and a greater chance that the true value of the slope coefficient falls outside of the interval). A smaller sample size or smaller variance in x will lead to a larger standard error for s_{b_2} and therefore a wider confidence interval.

13. The ordinary least squares estimator for β_2 (the one we have used in class and in Excel):
    (a) Minimizes the sum of $(y_i - \hat{y}_i)$.
    (b) Minimizes the sum of $(y_i - \bar{y})$.
    (c) Minimizes the sum of $(y_i - \hat{y}_i)^2$.
    (d) Minimizes the sum of $(y_i - \bar{y})^2$.

    (c) The ordinary least squares estimator minimizes the sum of the squared residuals.

Use the following information to answer questions (14) and (15). Suppose that we regress the number of races a marathon runner wins on the average time it takes the runner to finish a marathon in minutes and get the estimates for the intercept and slope coefficient, b_1 and b_2 respectively. Now we run the same regression but measure the average finishing time in hours instead of minutes and get a new set of estimated coefficients $\tilde{b_1}$ and $\tilde{b_2}$.

14. Which of the following will be true?
    (a) $b_2 = \tilde{b_2}$.
    (b) $|b_2| > |\tilde{b_2}|$.
    (c) $|b_2| < |\tilde{b_2}|$.
    (d) Not enough information.

    (c) Think of the formula we use to calculate b_2. All of the x_i values are getting divided by 60. This will lead to division of the numerator by 60 and division of the denominator by 60^2. The net effect is to multiply the old value of b_2 by 60.

15. Which of the following will be true?
    (a) $b_1 = \tilde{b_1}$.
    (b) $|b_1| > |\tilde{b_1}|$.
    (c) $|b_1| < |\tilde{b_1}|$.
    (d) Not enough information.

    (a) Recall that $b_1 = \bar{y} - b_2\bar{x}$. Our $\bar{x}$ is now $\frac{1}{60}$ times the old $\bar{x}$. However, b_2 is now 60 times larger than it was before. This multipliers will cancel each other out and leave our value of b_1 unchanged.

16. Suppose that for a given sample, the sample variance of y is 400, the sample variance of x is 100 and the slope coefficient from regressing y on x is 1. What is the correlation between x and y for this sample?
    (a) 1.
    (b) $\frac{2}{5}$.
    (c) $\frac{1}{2}$.
    (d) It depends on the estimated intercept from the regression.

    (c) The slope coefficient is equal to the correlation times the square root of the sample variance of y divided by the sample variance of x. So we have $1 = r_{xy}\sqrt{\frac{400}{100}}$.

17. Suppose that days of exercise increase early in life and then decrease later in life. If you had data on exercise and age and wanted to model the relationship between exercise and age and estimate it using ordinary least squares, what data transformations would you use?

    (a) Take the natural log of exercise.
    (b) Take the natural log of age.
    (c) Take the natural log of exercise and the natural log of age.
    (d) Use a polynomial in age.

    (d) The pattern described in the question is a u-shaped curve. The best way to fit this sort of curve is with a polynomial of our x variable which is age in this case.

18. The residual for a particular observation (x_i, y_i) is the:

    (a) The vertical distance between (x_i, y_i) and the regression line.
    (b) The vertical distance between (x_i, y_i) and $(\bar{x}, \bar{y})$.
    (c) The horizontal distance between (x_i, y_i) and the regression line.
    (d) The horizontal distance between (x_i, y_i) and $(\bar{x}, \bar{y})$.

    (a) The residual is just $y_i - \hat{y}_i$. This is the vertical distance between the data point and the regression line.

19. Which of the following is not an assumption we made when doing bivariate statistical inference?

    (a) The population model relating y to x is a linear function.
    (b) The error has mean zero.
    (c) The variance of the error terms is zero if the sample size is large.
    (d) The errors for different observations are unrelated.

    (c) We assumed that there was a constant variance for the error terms.

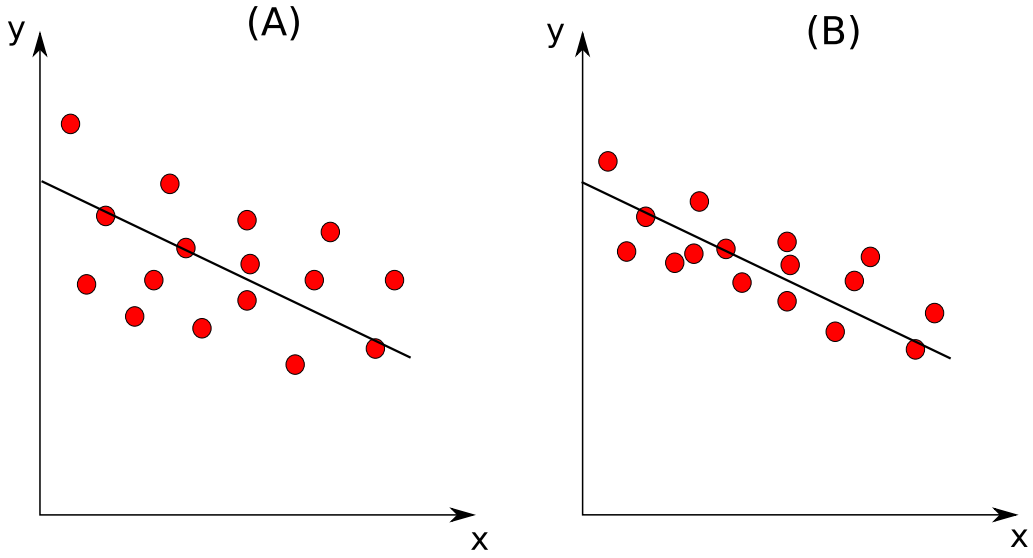
20. Dummy variables are used when:

    (a) The scale of the independent variables is very different for different observations.
    (b) There is an exponential relationship between x and y.
    (c) One of our variables of interest is a categorical variable.
    (d) We have a smooth but nonlinear relationship between x and y.

    (c) Dummy variables give us a way to recode a categorical variable into a binary variable that can be included in a regression.

SECTION II: SHORT ANSWER (40 points)

1. (8 points) On the graphs below, sketch two scatter plots with regression lines (you do not need to include any numbers on your graphs). You should include at least ten data points on each scatter plot. The two graphs should represent data that would give the same estimated slope coefficient and estimated intercept, have the same sample range for x, and have a negative covariance between y and x. The one difference is that the regression line for graph A should have a lower R^2 than the regression line for graph B.



2. (14 points) The following descriptive statistics were calculated in Excel from a dataset of annual GDP growth rates and unemployment rates for the United States. The sample consists of annual observations from the year 1960 to the year 2007. The first two tables contain descriptive statistics for the unemployment rate and the growth rate of GDP, respectively. The third table contains correlations between the variables.

unemployment rate	
Mean	5.84
Standard Error	0.21
Median	5.6
Mode	5.5
Standard Deviation	1.42
Sample Variance	2.03
Kurtosis	0.59
Skewness	0.73
Range	6.2
Minimum	3.5
Maximum	9.7
Sum	280.4
Count	48

gdp growth rate	
Mean	3.31
Standard Error	0.29
Median	3.45
Mode	2.5
Standard Deviation	1.99
Sample Variance	3.98
Kurtosis	0.15
Skewness	-0.48
Range	9.1
Minimum	-1.9
Maximum	7.2
Sum	158.8
Count	48

	unemployment rate	*gdp growth rate*
unemployment rate	1	
gdp growth rate	-0.26	1

Suppose that you want to estimate the following relationship:

$$g_t = \beta_1 + \beta_2 urate_t + \varepsilon_t \tag{1}$$

where g_t is the growth rate of GDP in year t, $urate_t$ is the unemployment rate in year t and ε_t is an error term.

(a) List two properties that the error terms should have if we want to get unbiased, consistent estimates β_1 and β_2 using ordinary least squares.

- The error should have mean zero.
- The error should be unrelated with the unemployment rate.
- The errors for different observations should have constant variance.
- The errors for different observations should be unrelated.
- The errors should be normally distributed.

(b) Calculate the value of the slope coefficient, b_2, you would get from regressing the growth rate of GDP on the unemployment rate. Explain in words what the meaning of this value is.

To get the slope coefficient we can use the information provided about the variances of the growth rate of GDP and the unemployment rate and the correlation between the two:

$$b_2 = r_{g,urate}\sqrt{\frac{s_{g,g}}{s_{u,u}}} = -.26 \cdot \sqrt{\frac{3.98}{2.03}} = -.36$$

This coefficient tells us that a one percent point increase in the unemployment

rate is associated with a .36 percent point decrease in the annual growth rate of GDP.

(c) Calculate the value of the intercept, b_1, you would get from regressing the growth rate of GDP on the unemployment rate. Explain in words what the meaning of this value is.

We can calculate the value of b_1 using the information on the means of the growth rate of GDP and the unemployment rate given in the tables and the value of b_2 we calculated above:

$$b_1 = \bar{g} - b_2 \bar{urate} = 3.31 + .36 \cdot 5.84 = 5.41$$

We can interpret this value as the predicted annual growth rate of GDP if there was no unemployment. If the unemployment rate was zero, the predicted annual growth rate of GDP would be 5.41%.

3. (18 points) Below is the regression output from regressing the number of prisoners per 100,000 people (*PRISON*) on the number of police officers per 100,000 people (*POLICE*). The unit of observation is a state (there is an observation for each of the 50 states and one observation for the District of Columbia).

Regression Statistics	
R Square	0.7041291
Standard Error	97.9046561
Observations	51

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-112.14	39.82	-2.82	0.00698	-192.15	-32.12
POLICE	1.40	0.13	10.8	1.48E-14	1.14	1.66

(a) Explain what the R^2 for this regression is telling us.

The R^2 is telling us that 70% of the variation in prisoners per 100,000 people across states is explained by the observed variation in the number of police officers per 100,000 people.

(b) Is there a statistically significant positive relationship between the number of police per 100,000 people and the number of prisoners per 100,000 people at a 5% significance level? Justify your answer.

There is a statistically significant relationship. The coefficient on police is positive and has a p-value that is less than .05.

(c) Use an upper one-tailed test and a 1% significance level to test whether the slope coefficient is greater than 1. Be certain to clearly state your null and alternative hypotheses, show your calculations, and clearly state your conclusions.

The null and alternative hypotheses are:

$$H_o: \ \beta_2 \leq 1$$

$$H_a: \ \beta_2 > 1$$

Our test statistic is:

$$t^* = \frac{1.40 - 1}{.13} = 3.08$$

Our cricital value is:

$$t_{.01,49} = TINV(.02, 49) = 2.40$$

Our test statistic is greater than the critical value which means that we will reject the null hypothesis that the coefficient is less than or equal to one in favor of the alternative hypothesis that the coefficient is greater than one.

(d) Explain one reason that we may expect the direction of causality to be from the number of police to the number of prisoners (that is, an increase in the number of police would cause an increase in the number of prisoners).

If we increase the number of police, more criminals will be caught leading to larger numbers of prisoners. So an increase in the number of police will cause an increase in arrests and the number of prisoners.

(e) Explain one reason that we may expect the direction of causality to go in the other direction.

If we have more prisoners, people may view crime as a major problem and increase the number of police officers. So an increase in the number of prisoners will cause an increase in funding for police increasing the number of officers.