Econ 362 - Government Regulation of Business
College of William and Mary
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Midterm 1 - Solutions

You have until 3:20pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Non-graphing calculators may be used (no graphing calculators or phones can be used). You may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can produce fractions of units and charge non-integer prices (so a firm could produce 82.4 units and sell at a price of \$5.325 per unit). Remember to put your name on the exam. Good luck!

Name: **ID Number:**

1. (15 points) One issue with collusion from a firm's perspective is that it is hard to keep other firms from breaking the agreement. What features of a collusive agreement would help ensure that the other firms keep their word? (It is useful to think about the common features of the examples discussed in class). In what types of markets would collusion be most likely to take place without firms breaking the collusive agreement? Be certain to explain your answer.

   There are a variety of things you could discuss in your answer. Among the key issues that your answer should address are the following:

   - The actions of each firm need to be easily monitored by the other firms so that firms can confirm that everyone is following the agreement. In the cases we saw in class this typically meant sharing data on sales to customers, publicly publishing price schedules and so on.
   - Another key feature would be the incorporation of a punishment strategy that will prevent individual firms from cheating. A punishment strategy could be something along the lines of saying that firms will revert to the perfectly competitive price if one firm cheats. A well designed punishment strategy would include a punishment that outweighs the gains from cheating but is also credible in the sense that firms will go through with the punishment if one firm cheats. For example, firms claiming that they will sell below cost for every period after one firm cheats is not credible. After the single firm cheats, the other firms would not follow through with their threat; it would lead to negative profits for them from that point on.
   - A final feature that will help ensure collusion works is to explicitly forbid firms from engaging in other competitive behavior that would undermine the collusive agreement. In the Sotheby's case discussed in class, this included establishing conditions that prevented the two auction houses from offering loans to their customers or offering to make charitable contributions on behalf of customers. These would be ways to compete for customers that would undermine the collusive agreement on seller commissions.

   As for the types of markets where collusion would be most likely to take place, these would be markets in which the gains from collusion are substantial, the number

of firms is small (so the colluding firms could influence market outcomes), and the actions of firms would be easily observable to one another (for example, firms sticking to agreed upon prices is easy to confirm in an industry where price schedules are published but hard to confirm in a scenario where firms submit sealed bids for contracts).

2. (25 points) Two farmers are selling squash at the Williamsburg farmers market. Farmer A is from Williamsburg and each additional squash he brings to the market raises his total costs by \$2. Farmer B is from Richmond and each additional squash he brings to the market increases his costs by \$5 due to the higher transportation costs. Demand for squash at the farmers market is given by:

$$D(p) = 500 - 50p \tag{1}$$

   (a) Write down the total cost and marginal cost functions ($C_A(S_A)$ and $MC_A(S_A)$) for farmer A in terms of the number of squash he brings to the market, S_A. Write down the total cost functions for farmer B as well ($C_B(S_B)$ and $MC_B(S_B)$). You can assume that fixed costs for both farmers are zero.

   Given that there are no fixed costs, the farmers' total costs come entirely from the marginal costs transported. Since the marginal costs are constant in this case, total costs will simply be marginal cost multiplied by quantity:

$$C_A(S_A) = 2S_A$$
$$MC_A(S_A) = 2$$
$$C_B(S_B) = 5S_B$$
$$MC_B(S_B) = 5$$

   (b) What is the efficient quantity of squash sold at the farmers market?

   For efficiency we want the greatest total surplus possible on each squash sold. Notice that for any given squash, farmer A can provide it at lower costs than farmer B. So the difference between the consumer's marginal benefit on a squash and the marginal cost of providing that squash will always be bigger if farmer A provides the squash. To maximize total surplus we therefore want farmer A to supply all of the squash. Every squash up to the point where price is equal to farmer A's marginal cost will create additional total surplus (every squash after that would reduce total surplus since MC would be greater than MB). So the efficient quantity is given by:

$$p(S_{eff}) = MC_A(y_{eff})$$

   To get the inverse demand function $p(S)$ we simply need to rearrange the demand equation:

$$D(p) = 500 - 50p$$
$$S = 500 - 50p$$
$$p = 10 - \frac{1}{50}S$$

   Plugging this and the marginal costs of firm A back into our condition for the efficient quantity gives us:

$$10 - \frac{1}{50}S_{eff} = 2$$
$$\frac{1}{50}S_{eff} = 8$$
$$S_{eff} = 400$$

(c) Assume the farmers compete on price. In other words, they each announce their price and the farmer announcing the lower price gets all of the demand at that price. If they announce the same price, they split the demand evenly. If the farmers can only choose integer prices ($5, $4, and so on but not $5.50 or $4.50), what will the equilibrium price be?

The farmer's will undercut each other down to a price of $5, the marginal cost of farmer B. At this point, farmer B can no longer drop his price without losing money. The main question is whether farmer A will then drop his price down to $4 to capture the entire market instead of splitting the market with farmer B. To decide whether he would do this, we can check is profits under each scenario:

$$\pi_{p=5} = p \cdot \frac{D(p)}{2} - C_A(\frac{D(p)}{2})$$

$$\pi_{p=5} = 5 \cdot \frac{500 - 50 \cdot 5}{2} - 2 \cdot \frac{500 - 50 \cdot 5}{2}$$

$$\pi_{p=5} = 375$$

$$\pi_{p=4} = p \cdot D(p) - C_A(D(p))$$

$$\pi_{p=4} = 4 \cdot (500 - 50 \cdot 4) - 2 \cdot (500 - 50 \cdot 4)$$

$$\pi_{p=4} = 600$$

So farmer A will drop his price to $4 since it generates greater profits. Note that this will not always be true. If the discrete drop in price is large enough, it is possible that the reduction in total profits by cutting price is large enough that a firm is better off getting half of the larger profits rather than all of the smaller profits. It is therefore necessary in a case like this with discrete prices to check profits at the two different prices.

(d) Calculate consumer surplus, producer surplus for farmer A and producer surplus for farmer B in the equilibrium outcome from part (c).

From above, we know that farmer A's profits will be $600. Given that he has no fixed costs, this will also be his producer surplus. Farmer B does not produce anything so his producer surplus is zero. For consumer surplus, we simply need to calculate the area under the demand curve above the price found in part (c). Noting that the inverse demand curve has a vertical intercept of 10, this area is:

$$CS = \frac{1}{2}(10 - p)(D(p) - 0)$$

$$CS = \frac{1}{2}(10 - 4)(300 - 0)$$

$$CS = 900$$

(e) Suppose that the Virginia legislature is considering subsidizing transporation costs for farmers to get their produce to farmers markets. This would lead to the state government paying farmer B $1 for each squash he transports to the farmers market. Farmer A would not get a subsidy because he is already located in Williamsburg. With this subsidy in place, what is the new equilibrium price for squash?

The subsidy would effectively lower farmer B's marginal costs to \$4. Now the farmers would undercut each other down to \$4 and farmer A would have to decide whether to share the profits at a price of \$4 or lower his price to \$3. We have already found above that farmer A's profits selling the total quantity at \$4 would be \$600. When splitting demand evenly, farmer A would get half of those total profits or \$300. Now we just need to check the profits farmer A would earn at a price of \$3:

$$\pi_{p=3} = p \cdot D(p) - C_A(D(p))$$

$$\pi_{p=3} = 3 \cdot (500 - 50 \cdot 3) - 2 \cdot (500 - 50 \cdot 3)$$

$$\pi_{p=3} = 350$$

So farmer A still earns higher profits by pricing below farmer B to capture the entire market, making the new equilibrium price \$3.

(f) Should the state government proceed with passing this legislation? Your answer should address whether the benefits of the subsidy exceed the costs to the state government and should be supported by numerical evidence.

Notice that in the new equilibrium, the new consumer surplus is:

$$CS = \frac{1}{2}(10 - p)(D(p) - 0)$$

$$CS = \frac{1}{2}(10 - 3)(350 - 0)$$

$$CS = 1225$$

So total surplus is now:

$$TS_{p=3} = CS + PS_A + PS_B$$

$$TS_{p=3} = 1225 + 350 + 0$$

$$TS_{p=3} = 1575$$

Total surplus before the subsidy was:

$$TS_{p=4} = CS + PS_A + PS_B$$

$$TS_{p=4} = 900 + 600 + 0$$

$$TS_{p=4} = 1500$$

So total surplus has increased by \$75. This is the benefit of the proposed regulation. As far as the costs go, they are equal to the amount the state spends on the subsidy. In equilibrium, farmer B does not supply any squash to the market so the state spends nothing on the subsidy. Clearly then the benefits exceed the costs and the regulation should be put in place. (Note: You may argue that the state is paying subsidies during however many periods it takes to move from the old equilibrium to the new one. However, note that the subsidy

would be no larger than \$150 (half of total output at the original price of \$4 is 150) in any single period and the increase in total surplus of \$75 will exist for every period after equilibrium is reached. Therefore it still seems that if the new equilibrium is reached in any reasonable number of periods the benefits will outweight the costs.)

3. (25 points) Think about the Cournot duopoly model we discussed in class in which firms compete on quantity. This model can be extended to include additional firms. Suppose that we have three identical firms producing hats. Each individual firm i has the following total cost and marginal cost functions

$$C_i(y_i) = 5y_i \tag{2}$$

$$MC_i(y_i) = 5 \tag{3}$$

where y_i is firm i's output (so total output is $\sum_{i=1}^{3} y_i$). The inverse demand curve for hats is given by:

$$p(y) = 10 - \frac{1}{100}y \tag{4}$$

(a) Write down firm 1's profits as a function of firm 1's output, y_1, and the outputs of the other two firms $(y_2 + y_3)$.

Firm 1's profits will be a function of its output and of the market price which will be a function of total output:

$$\pi_1(y_1) = p(y_1 + y_2 + y_3) \cdot y_1 - C_1(y_1)$$

$$\pi_1(y_1) = (10 - \frac{1}{100}(y_1 + y_2 + y_3)) \cdot y_1 - 5y_1$$

$$\pi_1(y_1) = 10y_1 - \frac{1}{100}y_1^2 - \frac{1}{100}(y_2 + y_3)y_1 - 5y_1$$

$$\pi_1(y_1) = 5y_1 - \frac{1}{100}y_1^2 - \frac{1}{100}(y_2 + y_3)y_1$$

(b) Find an expression for firm 1's marginal revenue as a function of $y_2 + y_3$.

Total revenue for firm is just price times firm 1's output where price is a function of total output:

$$R_1(y_1) = p(y_1 + y_2 + y_3) \cdot y_1$$

$$R_1(y_1) = (10 - \frac{1}{100}(y_1 + y_2 + y_3)) \cdot y_1$$

$$R_1(y_1) = 10y_1 - \frac{1}{100}y_1^2 - \frac{1}{100}(y_2 + y_3)y_1$$

Marginal revenue is the derivative of this function with respect to firm 1's output treating the other firms' outputs as constant:

$$MR_1(y_1) = \frac{dR_1(y_1)}{dy_1} = 10 - \frac{1}{50}y_1 - \frac{1}{100}(y_2 + y_3)$$

(c) Given your answer to part (b), find an expression for firm 1's profit maximizing output as a function of $y_2 + y_3$.

Like any profit maximizing firm, firm 1 will want to produce up to the point where marginal revenue equals marginal cost. If they produce less than this, marginal revenue will be greater than marginal costs implying that producing an additional unit would lead to a greater increase in revenue than in costs, increasing profits. If they produce more than this, marginal costs would exceed marginal revenue implying that costs exceeded revenue on the last unit produced leading to a reduction in profits. Setting marginal revenue equal to marginal cost gives us:

$$MR_1(y_1) = MC_1(y_1)$$
$$10 - \frac{1}{50}y_1 - \frac{1}{100}(y_2 + y_3) = 5$$
$$\frac{1}{50}y_1 = 5 - \frac{1}{100}(y_2 + y_3)$$
$$y_1 = 250 - \frac{1}{2}(y_2 + y_3)$$

(d) Assume that since all of the firms are identical, each produces $\frac{y}{3}$ if total output is y. If this is true, what will be the equilibrium output per firm and the equilibrium price?

We can find the equilibrium quantity by substituting $\frac{y}{3}$ into the above equation for each firm's output:

$$\frac{y}{3} = 250 - \frac{1}{2}(\frac{y}{3} + \frac{y}{3})$$
$$\frac{y}{3} = 250 - \frac{y}{3}$$
$$\frac{2}{3}y = 250$$
$$y = 375$$

Plugging this quantity into the inverse demand function will give us the equilibrium price:

$$p(375) = 10 - \frac{1}{100} \cdot 375$$
$$p(375) = 6.25$$

So the equilibrium quantity will be 375 hats and the equilibrium price will be \$6.25. Notice that this equilibrium price is above the marginal cost of the firms but below the monopoly price.

(e) What would the equilibrium output per firm and the equilibrium price be if there were four identical firms instead of three? (Hint: You should not have to redo all of your work, just notice how the number of firms entered into your equations.)

Look back at the marginal revenue of firm 1 and think about how the number of firms influenced it. The number of firms simply came in in the last term as the sum of the other firms' outputs. We could generalize this to having n total firms in the following way:

$$MR_1(y_1) = 10 - \frac{1}{50}y_1 - \frac{1}{100}(\sum_{i=2}^{n} y_i)$$

Where we had the sum of firm 2 and firm 3's outputs we now have the sum of outputs from firm 2 all the way up to firm n. Firm 1 will still set marginal revenue equal to marginal cost:

$$10 - \frac{1}{50}y_1 - \frac{1}{100}\sum_{i=2}^{n} y_i = 5$$

leading to the profit maximizing quantity of:

$$y_1 = 250 - \frac{1}{2}\sum_{i=2}^{n} y_i$$

Once again assuming that the firms produce the same amount in equilibrium, we can substitute $\frac{y}{n}$ for each y_i in the above equation to get the equilibrium quantity:

$$\frac{y}{n} = 250 - \frac{1}{2}(\sum_{i=1}^{n} \frac{y}{n})$$

$$\frac{y}{n} = 250 - \frac{1}{2}(n-1)\frac{y}{n}$$

$$(1 + \frac{n-1}{2})\frac{y}{n} = 250$$

$$\frac{1+n}{2n}y = 250$$

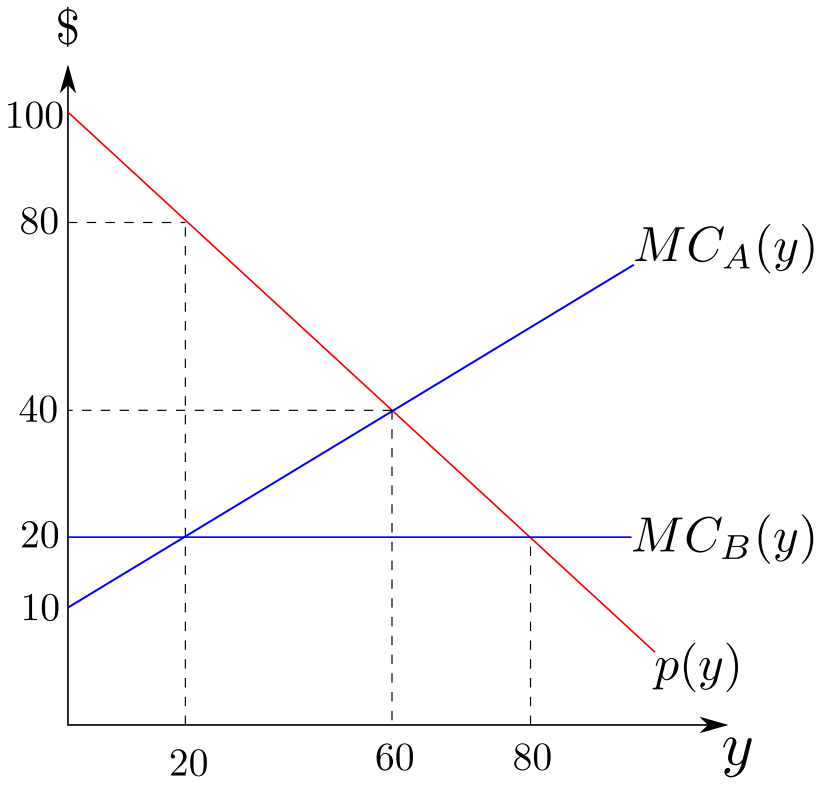
$$y = \frac{2n}{1+n}250$$

Plugging in four for n in this equation would give us an equilibrium quantity of 400. Plugging this quantity into the inverse demand function would give us an equilibrium price of \$6. Note that this is much more work than is required for full credit. If you had simply noted that y_1 should be switched to $\frac{y}{4}$ and instead of having $y_2 + y_3$ being $\frac{2}{3}y$ is switched to $y_2 + y_3 + y_4$ being $\frac{3}{4}y$ you would have arrived at the same answer and received full credit.

(f) The local government is considering running an ad campaign to encourage more firms to enter the industry. What information would you need to determine whether this should be done? Be as specific as possible. Assume the only benefits the government cares about are increases in total surplus.

Note from the previous part that increasing the number of firms moves us closer to the efficient quantity of 500 hats. To determine whether the ad campaign is worthwhile, the government would need to determine how many firms would be induced to enter the industry as a result of the campaign, calculate the increase in total surplus that results from these firms entering and pushing equilibrium quantity toward the efficient quantity, and then compare this gain in total surplus to the cost of the ad campaign. If the increase in total surplus is greater than the cost of the ads, the ads should be run.

4. (20 points) The graph below shows the demand curve for a particular market served by a monopolist. Also shown are two different marginal cost curves for the monopolist corresponding to two different production technologies, technology A and technology B. Use the graph to answer the questions below.

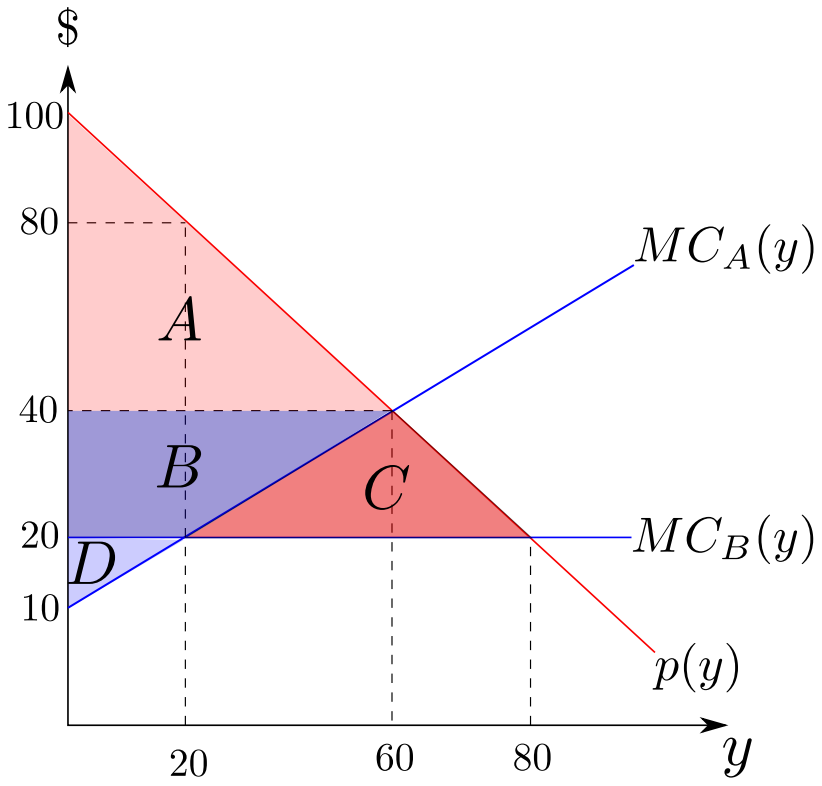


(a) Suppose that a regulator can force the monopolist to produce at the efficient quantity (based on the monopolists chosen technology) and charge the price consumers are willing to pay at that quantity. If the monopolist can choose which production technology it wants to use, which will it choose? Use numerical evidence to justify your answer.

Note that the efficient quantity depends on the chosen technology. If technology A is used, total surplus will be maximized by producing where MC_A intersects the demand curve. This occurs at a quantity of 60 units. If technology B is used, total surplus will be maximized at a quantity of 80 units. If the monopolist chooses to use technology A and therefore is forced to produce at 60 units, the monopolist will have producer surplus equal to the area above MC_A and below the price up to 60 units. This producer surplus is equal to area $B + D$ on the graph below:

$$PS_A = \frac{1}{2}(40 - 10)(60 - 0)$$
$$PS_A = 900$$

If technology B is chosen, the firm faces constant marginal costs equal to price meaning that they get zero producer surplus on each unit. Clearly the firm will prefer using technology A and receiving \$900 in producer surplus to using technology B and receiving zero producer surplus.



(b) Suppose instead that the regulator can choose which technology the monopolist uses. If the regulator is concerned with maximizing total surplus, which technology will the regulator choose? Use numerical evidence to support your answer.

Once again, refer to the graph above to see the relevant areas for calculating consumer and producer surplus under the two different technologies. Consumer surplus under technology A will be equal to area A while producer surplus will be equal to area $B + D$. Total surplus will be the sum of these:

$$TS_A = CS_A + PS_A$$

$$TS_A = \frac{1}{2}(100 - 40)(60 - 0) + \frac{1}{2}(40 - 10)(60 - 0)$$

$$TS_A = 2700$$

Under technology B, the producer surplus will be zero but the consumer surplus will be area $A + B + C$:

$$TS_B = CS_B + PS_B$$

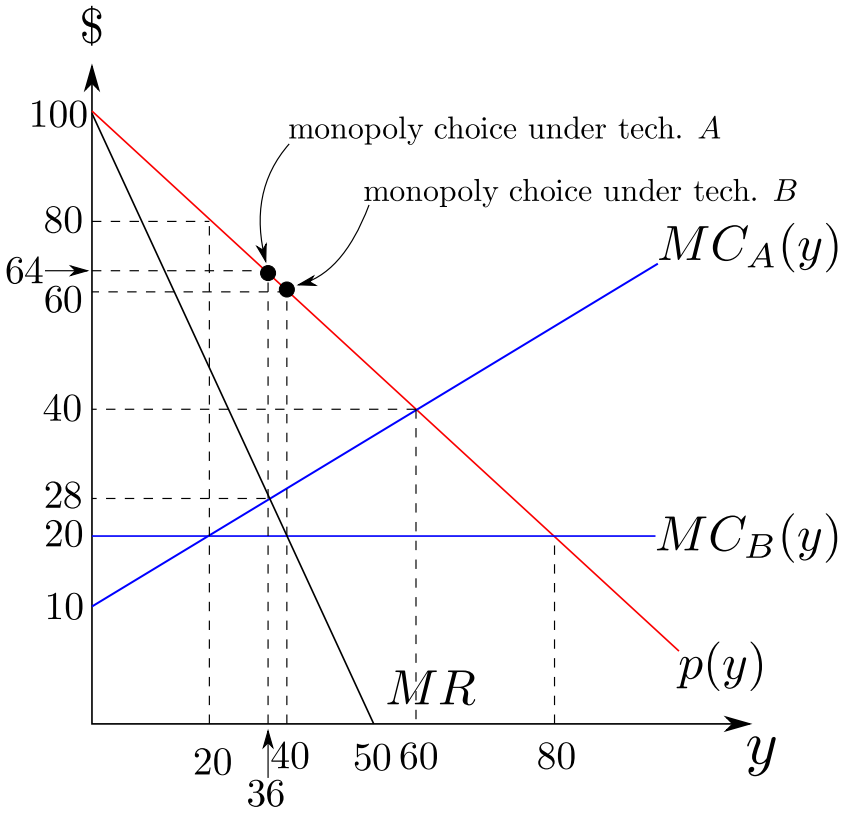
$$TS_B = \frac{1}{2}(100 - 20)(80 - 0) + 0$$

$$TS_B = 3200$$

So total surplus will be greater under technology B, leading the regulator to choose that technology.

(c) Now suppose that the regulator cannot force the monopolist to produce at a particular quantity. The monopolist is free to set its own prices and quantity. On the graph, label the level of output the firm will choose if it uses technology A and the level of ouput it would choose if it uses technology B.

The monopolist will choose to produce at the quantity where marginal cost equals marginal revenue. The marginal revenue curve will be a curve twice as steep as the demand curve but with the same intercept. Given the original graph, this means that the marginal revenue curve has a vertical intercept of 100 and a slope of -2. Adding this curve to the graph, we can see the monpoly quantities under each technology. These quantities are shown on the graph below. (Note that you should be able to calculate these exact values from the information on the graph. Having multiple points on each line lets you identify the equation for each line: $MC_A(y) = 10 + \frac{1}{2}y$, $MC_B(y) = 20$, $p(y) = 100 - y$, $MR(y) = 100 - 2y$. With these equations, it is straightfoward to solve for each point of intersection.)



(d) Still assuming the monopolist can set its own price and quantity, which technology will the monopolist choose? Is there any deadweight loss generated by letting the monopolist choose the technology rather than the regulator in this case? Use numerical evidence to support your answers.

The monopolist's producer surplus under technology A will be the area above the marginal cost curve below the price up to the quantity at which the marginal cost curve intersects the marginal revenue curve:

$$PS_A = \frac{1}{2}(28-10)(36-0) + (64-28)(36-0)$$

$$PS_A = 1620$$

Under technology B, the producer surplus is no longer zero since the monopolist is no longer required to charge a price equal to marginal cost. The producer surplus under technology B is now:

$$PS_B = (40-0)(60-20)$$

$$PS_B = 1600$$

The producer surplus is slightly higher under technology A so the monopolist will choose that technology over technology B. Total surplus under technology A is:

$$TS_A = CS_A + PS_A$$

$$TS_A = \frac{1}{2}(100-64)(36-0) + 1620$$

$$TS_A = 2268$$

Total surplus under technology B is:

$$TS_B = CS_B + PS_B$$

$$TS_B = \frac{1}{2}(100-60)(40-0) + 1600$$

$$TS_B = 2400$$

So technology B leads to greater total surplus. By choosing technology A, the monopolist generates an additional deadweight loss of \$132 relative to choosing technology B.

5. (15 points) Suppose that Movie Tavern and New Town are the only two places to watch movies in Williamsburg. They have the same constant marginal costs per movie ticket. Currently, the two theaters compete on price leading to the equilibrium price being equal to marginal cost. Movie Tavern decides to engage in predatory pricing to drive New Town out of business. Movie Tavern prices each ticket to be half of the marginal cost of the ticket. Assume that Movie Tavern's approach works. After ten time periods, New Town shuts down and Movie Tavern is a monopoly from that point on.

   (a) Will this predatory pricing scheme be beneficial or harmful to consumers? Fully explain your answer. Be certain to consider consumers' discount rate (interest rate) when answering.

   During the periods in which Movie Tavern is pricing below cost, consumers will get lower prices and buy a greater quantity of tickets than they would in the competitive outcome. This will lead to an increase in consumer surplus. However, after New Town exits the market, consumers will face higher prices and buy lower quantities of tickets than in the competitive outcome leading to a decrease in consumer surplus. Whether the overall net benefit to consumers is positive or negative depends on how heavily consumers discount future benefits or costs. If consumers have a high discount rate, this will reduce the present value of the future loss in consumer surplus possibly making the net effect of the predatory pricing positive. However, if consumers have a low discount rate the present value of these future periods of lost consumer surplus will be larger possibly leading to a net loss for consumers.

   (b) Explain why the predatory pricing will lead to inefficient outcomes. Be certain to consider both the periods during which Movie Tavern prices below cost and the periods during which it is a monopoly.

   The predatory pricing leads to inefficient outcomes both during the periods where price is below marginal cost and in the periods where Movie Tavern is a monopoly. In the initial periods, by pricing below marginal cost the last few tickets purchased by consumers will provide a marginal benefit to consumers that is below the marginal cost of the ticket. This means that the sale of the ticket generates a net loss to society. When Movie Tavern moves to the monopoly price, the price of a ticket will be above marginal cost meaning that there will be tickets for which marginal benefit to consumers is below the current price but above the marginal cost that will not be sold. These tickets would produce a net benefit to society. Not selling them generates a deadweight loss.