

Basic Rules

- We will have three different auctions with a \$2 bill being auctioned off in each one
- Each of you will receive \$2 in quarters and \$1 in dimes
- You may only use these coins in the auctions
- For example, suppose you are down to two quarters and one dime by the final auction, your possible bids are limited to \$0.10, \$0.25, \$0.35, \$0.50, and \$0.60
- You get to keep whatever money you have left at the end of the auctions (including any \$2 bills you win)

Auction 1 - First-Price Sealed Bid Auction

- Everyone writes down their bid on their index card
- Once all bids are in, I will look at the cards and reveal the highest bid
- The person with the highest bid wins the \$2 bill
- Ties will be broken with the flip of a coin
- The winning bidder pays the amount of his or her bid
- Everyone else pays nothing

Auction 2 - Third-Price Sealed Bid Auction

- Everyone writes down their bid on their index card
- Once all bids are in, I will look at the cards and reveal the highest bid
- The person with the highest bid wins the \$2 bill
- Ties will be broken with the flip of a coin
- The winning bidder pays the amount of the third highest bid
- Everyone else pays nothing
- Note on ties: Suppose the top bids are \$1.10, \$0.90, \$0.90, and \$0.80. The third highest bid in this case would be \$0.90

Auction 3 - All-Pay Sealed Bid Auction

- Everyone writes down their bid on their index card
- Once all bids are in, I will look at the cards and reveal the highest bid
- The person with the highest bid wins the \$2 bill
- All individuals pay the amount of their bid

Optimal Bidding in a Second-Price Sealed Bid Auction

- Let's say you value the object being auctioned at V_i
- You have three basic options about the bid you submit, B_i :
 - Overbid: $B_i > V_i$
 - Underbid: $B_i < V_i$
 - Bid your true value: $B_i = V_i$
- We'll write the maximum of all other bids as $\max_{j \neq i} B_j$
- The goal is to show that bidding your true value is a dominant strategy

Overbid: $B_i > V_i$

- Case 1: $\max_{j \neq i} B_j > B_i$
- In this case, you lose the auction and get zero
- Note that this is a good thing since winning would force you to pay over V_i
- You would also lose the auction bidding $B_i = V_i$ so the two strategies lead to the same outcome

Optimal Bidding in a Second-Price Sealed Bid Auction

Overbid: $B_i > V_i$

- Case 2: $\max_{j \neq i} B_j < V_i < B_i$
- In this case, you win the auction and get $V_i - \max_{j \neq i} B_j$ which is positive
- You would also win the auction bidding $B_i = V_i$ and get the same $V_i - \max_{j \neq i} B_j$
- Once again, the two strategies lead to the same outcome

Optimal Bidding in a Second-Price Sealed Bid Auction

Overbid: $B_i > V_i$

- Case 3: $V_i < \max_{j \neq i} B_j < B_i$
- In this case, you win the auction and get $V_i - \max_{j \neq i} B_j$ which is negative
- If you bid $V_i = B_i$ you would lose the auction and get zero
- In this case, you are better off bidding your true value
- So bidding your true value always does at least as well as overbidding and in some cases does better making it a better strategy

Underbid: $B_i < V_i$

- Case 1: $\max_{j \neq i} B_j < B_i$
- In this case, you win the auction and get $V_i - \max_{j \neq i} B_j$ which is positive
- You would also win the auction bidding $B_i = V_i$ and get the same $V_i - \max_{j \neq i} B_j$
- So the two strategies lead to the same outcome

Underbid: $B_i < V_i$

- Case 2: $B_i < V_i < \max_{j \neq i} B_j$
- In this case, you lose the auction and get zero
- You would also lose the auction bidding $B_i = V_i$
- So the two strategies lead to the same outcome

Underbid: $B_i < V_i$

- Case 3: $B_i < \max_{j \neq i} B_j < V_i$
- In this case, you lose the auction and get zero
- If you bid $B_i = V_i$ you would win the auction and get $V_i - \max_{j \neq i} B_j$ which is positive
- In this case you are better off bidding your true value
- So bidding your true value always does at least as well as underbidding and in some cases does better

Optimal Bidding in a Second-Price Sealed Bid Auction

- So bidding your true value is a weakly dominant strategy relative to overbidding
- Bidding your true value is also a weakly dominant strategy relative to underbidding
- This makes bidding your true value the optimal strategy
- What about our first-price sealed bid auction?
- Overbidding is clearly a bad move
- As for underbidding, think about Case 1 and Case 3