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## Problem Set 5

This problem set will not be graded and does not need to be turned in. However, the problem set does cover topics that will be on the second midterm, so it is highly recommended that you work through the problems in the same way you would for a graded problem set. Solutions for this problem set are posted on Blackboard.

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1. **Using Multiple Factories** You are the owner of a car company that currently has two different factories, an old one and a new one. The old factory is uses an old technology that requires large amounts of labor to produce cars. The production technology for the old firm is given by  $f_{old}(L) = 5L^{\frac{1}{2}}$ . The new factory uses a much more modern production technology given by  $f_{new}(L) = 10L^{\frac{1}{2}}$ .
  - (a) What are costs as a function of output for each individual factory?
  - (b) Suppose that you want to produce 1000 cars. How many cars will you produce at the old factory and how many will you produce at the new factory?
  - (c) Derive an expression for the total cost of producing  $y$  cars (assuming that you always divide production between the factories in the optimal way). How does this cost function compare to the individual factory cost functions?
  - (d) It turns out that you can upgrade your old factory to make it just as efficient as the new factory. To pay for the upgrade, you can take out a loan that will be paid back in equal annual installments for the rest of the lifetime of your business. If you plan on producing 1000 cars every year, what is the largest annual payment you are willing to make to upgrade your old factory?

2. **Industry Supply** Suppose that there are two types of firms in a perfectly competitive market for widgets ( $w$ ). Firms of type  $A$  have costs given by  $C_A(w) = 5w^2 + 2w + 10$ . Firms of type  $B$  have costs given by  $C_B(w) = 3w^2 + 5$ . There are 100 firms of type  $A$  and 180 firms of type  $B$ .
- (a) What are the individual firm supply functions for each type type of firm ( $S_A(p)$  and  $S_B(p)$ )? Are there any prices at which no firms produce? Are there any prices at which some firms produce but others do not?
  - (b) What is the industry supply function? Graph the industry supply function and be certain to label any kinks and all relevant slopes.
  - (c) Suppose that in the long run, when firms can adjust all inputs, all firms have the following cost function:  $C(w) = w^3 - 20w^2 + 110w$ . The market demand for widgets is still given by  $D(p) = 1000 - p$ . What must the price of widgets be in the long run equilibrium? How many firms will there be producing widgets? (Hint: Remember that each individual firm should be earning zero profits in the long run.)