*Econ 303 - Intermediate Microeconomic Theory*
College of William & Mary March 12, 2013
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# Problem Set 4

This problem set will be graded and is due by **5pm** on **Friday, March 22nd** in my mailbox in the economics department. You may turn in problem sets early by putting them in my mailbox in the economics department or by dropping them off in lecture. No late problem sets will be accepted. You are welcome to work in groups. If working in a group, everyone in the group must still submit an individual problem set.

1. **Badly Behaved Production Functions**

   We typically work with well-behaved production functions where the marginal product of each input is diminishing and the technical rate of substitution is diminishing. In this problem, we'll see what can happen when we don't have these nice properties. Consider the following production technology that uses two inputs ($A$ and $B$) to produce widgets ($W$):

   $$W = f(A, B) = A^2 + B^2 \tag{1}$$

   (a) Determine which of the following properties hold for the production technology above (it may be helpful to graph several isoquants):

      i. Diminishing marginal product of $A$
      ii. Diminishing marginal product of $B$
      iii. Diminishing technical rate of substitution
      iv. Monotonicity
      v. Convexity

   (b) Let's assume that input $B$ is fixed in the short run at 5 units. The firm can vary the amount of input $A$ used. The price of a widget is \$10, the price of a unit of $A$ is \$2, and the price of a unit of $B$ is \$4. Graph the number of widgets produced as a function of the firm's choice of $A$. On the same graph, graph three different isoprofit lines. You should label the level of profits each isoprofit line corresponds to and the value for the intercept of the isoprofit line on the graph.

   (c) Given these prices and the fixed level of $B$, what is the value of $A$ the firm would choose and the level of profits they would earn if they use our profit-maximizing condition:

   $$p_A = p_W \cdot MP_A(A^*, \overline{B}) \tag{2}$$

   (d) Can the firm earn higher profits than what you found in part (c)? If it can, explain why our profit-maximizing condition did not work.

2. **Short Run vs. Long Run Costs**

   A firm uses labor ($L$) and machines ($M$) to produce output and has the following production function:

$$f(L, M) = 10L^{\frac{1}{2}}M^{\frac{1}{2}} \tag{3}$$

   The wage for a worker is \$25 and the price of a machine is \$100. In the short run, the firm's workers have unbreakable contracts so the firm cannot hire or fire workers but it can change the number of machines it uses. In the long run, both the number of workers and the number of machines can be adjusted. Currently the firm is employing 100 workers.

   (a) Suppose that the firm wants to produce $y$ units of output in the short run. Derive an expression for the number of machines the firm will need to use as a function of the desired level of output ($M(y)$).

   (b) Based on your answer to part (a), derive the costs for the firm as a function of output in the short run ($C_{SR}(y)$). Graph this cost function on a graph with output on the horizontal axis and costs on the vertical axis.

   (c) Derive expressions for the number of workers hired by the firm ($L(y)$) and the number of machines used by the firm ($M(y)$) in the long run if the firm wants to produce $y$ units of output and minimize costs.

   (d) Based on your answer to part (c), derive the long run cost function for the firm ($C_{LR}(y)$). Graph this function on the same graph as part (b). Find the value of output at the point where the two cost curves intersect. Based on your answer to part (c), how many workers will the firm hire at this level of output?

3. **Finding Firm Supply**

   Suppose that a firm's cost function is given by:

$$C(y) = \frac{1}{3}y^3 - 10y^2 + 105y + 5 \tag{4}$$

   (a) Derive expressions for $MC(y)$, $AC(y)$ and $AVC(y)$ and graph these three cost curves.

   (b) If the price of output is \$105, what quantity will the firm produce and what will the firm's profits be?

   (c) What is the lowest price at which the firm will produce output?

   (d) If the firm is producing output, what is the lowest level of output we will observe the firm producing?

   (e) Would your answers to (c) and (d) change if fixed costs were different?

4. **Building New Factories** A new car company is deciding to enter the US market and to produce all of their cars in the US. They need to decide how many factories to build. All of the factories would be identical, each having the following cost function: $C(y) = \frac{1}{3}y^3 - 10y^2 + 150y$.

   (a) Calculate the costs of producing 10 cars using one factory and compare this to the costs of producing 10 cars by splitting up production equally between two factories. Which way of producing 10 cars is more cost effective?

   (b) At what level of car production will the company be better off using two factories instead of one?

   (c) Can you find an expression for the optimal number of factories for this firm in terms of their desired level of output $y$? Given this result, how many cars are produced at each factory in terms of total output $y$? (Hint: Assume that since the factories are identical, output will be split equally between them. So the total costs of producing an amount $y$ using $n$ factories will be the cost of producing $\frac{1}{n}y$ at one factory times the number of factories.)

   (d) Find expressions for the marginal cost and average cost functions for a single factory. Graph these cost curves. On this graph, show where the optimal number of cars per factory is (the number you found in part (c)).