Econ 303 - Intermediate Microeconomic Theory
College of William and Mary
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Midterm 2 - Solutions

You have until 1:50pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can use fractions of units of inputs and produce fractions of units of output. Remember to put your name on the exam. Good luck!

Name:

1. (30 points) A firm uses wood (W) and metal (M) to make desks (D). Wood can be used in place of metal and metal can be used in place of wood. The marginal product of a sheet of wood is constant and equal to 10 (one sheet of wood produces 10 desks). The marginal product of a sheet of metal is constant and equal to 20 (one sheet of metal produces 20 desks).

   (a) Write down a production function for the firm giving the number of desks they can produce as a function of the sheets of metal and wood used ($f(M,W)$).

   Each input has a constant marginal product, so output rises linearly with each input. This tells us that the production function should be a linear function of M and W:

   $$f(M,W) = aM + bW$$

   The question now is what the values of the coefficients a and b should be. These coefficients are simply the marginal products of metal and wood, respectively. So our final production function is:

   $$f(M,W) = 20M + 10W$$

   (b) Given your production function from part (a), determine whether the firm's technology exhibits increasing, decreasing or constant returns to scale. Be certain to fully justify your answer.

   To test for the type of returns to scale, we can scale each input up by some arbitrary amount α and see whether output goes up by that same amount or by more or less.

   $$f(\alpha M, \alpha W) = 20(\alpha M) + 10(\alpha W)$$

   $$f(\alpha M, \alpha W) = \alpha 20M + \alpha 10W$$

   $$f(\alpha M, \alpha W) = \alpha(20M + 10W)$$

   $$f(\alpha M, \alpha W) = \alpha f(M,W)$$

   So when we scale up the inputs by any amount α, we scale up output by that exact same amount. This means that the technology exhibits constant returns to scale.

(c) Graph the isoquants for the firm corresponding to output levels of 20, 40, and 80 desks. Label all slopes and intercepts with numerical values if possible.

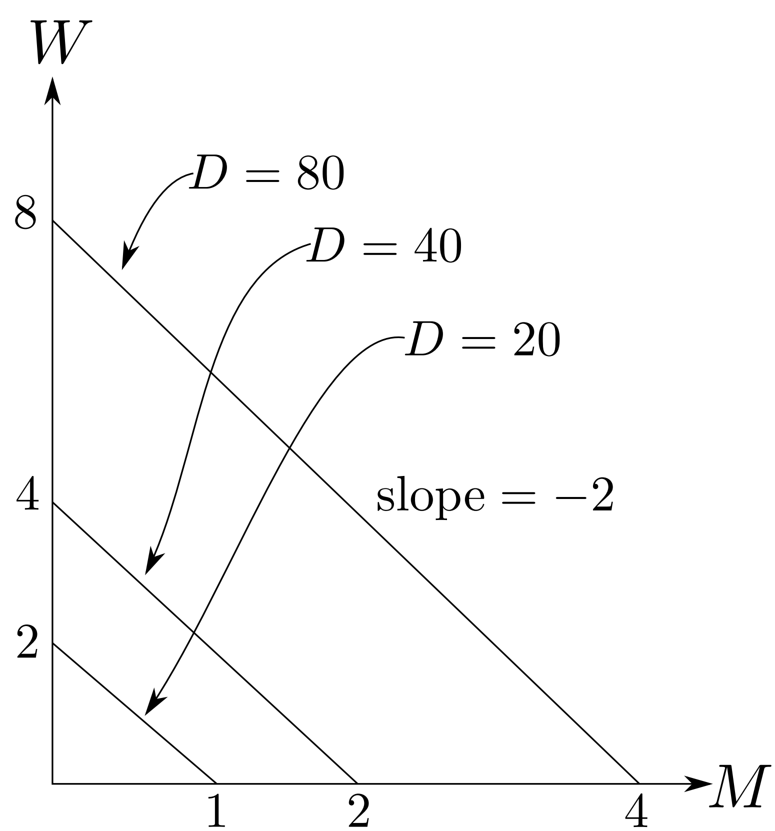
With a linear production function we will get linear isoquants. The slope of these isoquants on a graph with M on the horizontal axis and W on the vertical axis will be equal to the technical rate of substitution:

$$TRS = -\frac{MP_M}{MP_W}$$

$$TRS = -\frac{20}{10}$$

$$TRS = -2$$

This should make sense, each sheet of metal produces twice as many desks as each sheet of wood. So every time we use one more sheet of metal we can use two fewer sheets of wood and keep total output the same. The final step in graphing the isoquants is figuring out the appropriate endpoints. This is simply a matter of setting output at the appropriate value (either 20, 40 or 80 in this case) in the production function, plugging in zero for one input and solving for the value of the other input. The isoquants are shown in the graph below.



(d) Suppose that the price of a sheet of wood is \$100 and the price of a sheet of metal is \$150. Find an expression for the firm's minimum costs of producing D desks ($C(D)$). Be certain to clearly show how you arrived at your answer.

Normally we would try to find the point of tangency between the isoquant and the isocost line. However, in this case we have linear isoquants rather than our typical convex isoquants so there will be no point of tangency. Instead, we need to take a step back and think about which is the better input to use. Notice that the output from a dollar spent on metal is equal to:

$$\frac{MP_M}{p_M} = \frac{20}{150} = \frac{2}{15}$$

So every dollar spent on metal produces 0.13 desks. This will be true no matter how many sheets of metal we have already used because the marginal product of metal is constant. A dollar spent on wood will produce the following number of desks:

$$\frac{MP_W}{p_W} = \frac{10}{100} = \frac{1}{10}$$

So every dollar spent on wood produces 0.1 desks. Clearly using metal is more cost effective and will always be more cost effective regardless of how much wood or metal is being used since the marginal products of both are constant. Therefore the cheapest way to produce is to just use metal. This reduces are production function to:

$$D = f(M, 0) = 20 \cdot M + 10 \cdot 0 = 10W$$

Rearranging this gives us the amount of metal used as a function of the number of desks:

$$M(D) = \frac{D}{20}$$

Now, knowing that the amount of wood used will be zero and knowing the amount of metal as a function of D, we can write costs as a function of D:

$$C(D) = p_M M(D) + p_W W(D)$$

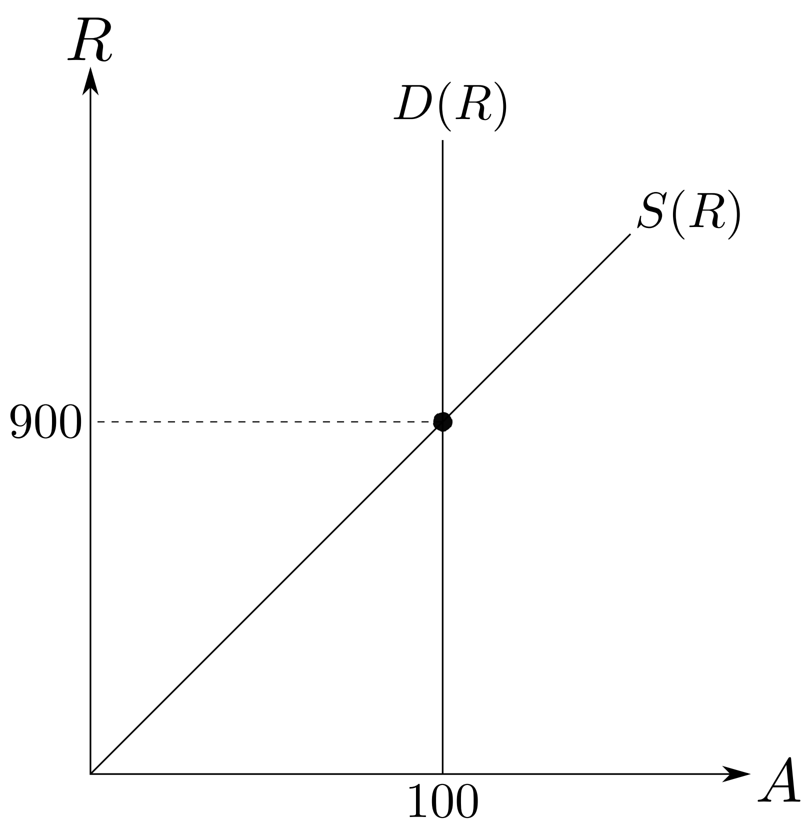
$$C(D) = 150 \cdot \frac{D}{20} + 100 \cdot 0$$

$$C(D) = \frac{15}{2} D$$

2. (20 points) Suppose that demand for apartments in Williamsburg is perfectly inelastic; everyone needs a place to live so they will pay whatever they need to for an apartment. The supply of apartments is not perfectly inelastic. If rents are higher, more apartments will be built and rented out. If rents are low, some apartments will be converted to other uses and no longer rented out. The supply curve for apartments is linear.

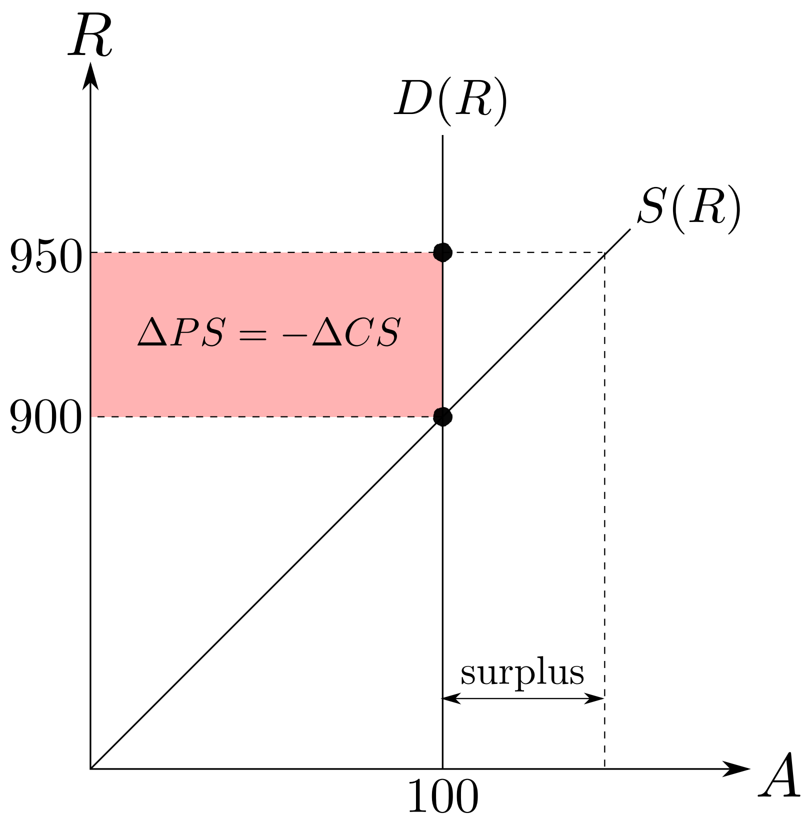
   (a) Suppose that the rental market is in equilibrium and the current market rent is \$900 with 100 apartments being rented out. Graph the supply and demand curves for apartments and show this equilibrium on the graph.

   The fact that demand is perfectly inelastic tells us that the demand curve will be a vertical line. Since the equilibrium quantity of apartments being rented out is 100, this vertical line will be at 100. The supply curve should be upward sloping, linear and intersect the demand curve at a rent of \$900. These curves are shown on the graph below.



   (b) Landlords lobby the City of Williamsburg to start controlling rents. They claim they cannot make enough money to survive and therefore need higher rents. The lobbying efforts succeed and the city imposes a rent floor of \$950, meaning that no rent can be set below \$950. Rents can be set higher than that. Show the effects of this rent floor on the equilibrium rent and number of apartments rented in Williamsburg on your graph. Label all relevant points with numerical values if possible.

Note that the equilibrium price was below \$950, so this rent floor will be binding. At a rent of \$950, people still demand 100 apartments so 100 apartments will be rented out. Landlords would like to rent out more (the supply of apartments at a price of \$950 is greater than 100) but they will have no one to rent to, so the equilibrium quantity remains at 100 while the equilibrium price has risen to \$950. There will be a surplus of apartments offered by landlords as shown on the graph below.



(c) Calculate the change in consumer surplus, change in producer surplus and the deadweight loss created by the rent floor.

For consumer surplus, note that the consumers are renting the same number of apartments as before only they are paying \$50 more in rent on each apartment, lowering the consumer surplus on each apartment by \$50. So the change in consumers surplus is:

$$\Delta CS = -50 \cdot 100 = -5000$$

This surplus lost by consumers is completely transferred to producers: the landlords are still renting 100 apartments only they are renting them at a price that is now \$50 higher. So producer surplus increases by \$5,000. There is no deadweight loss because total surplus has not changed. The same apartments are being rented out leading to the same total surplus, the only thing that has changed is how that surplus is distributed.

3. (30 points) A printer operating in a perfectly competitive industry faces the following total cost function for producing books:

$$C(B) = B^2 + 10B \tag{1}$$

where B is the number of books that the printer produces. Given this cost function, the marginal cost of producing a book is given by:

$$MC(B) = 2B + 10 \tag{2}$$

(a) Find expressions for the firm's average cost curve ($AC(B)$) and average variable cost curve ($AVC(B)$). Graph these curves along with the marginal cost curve. Label all intercepts and slopes with numerical values if possible.

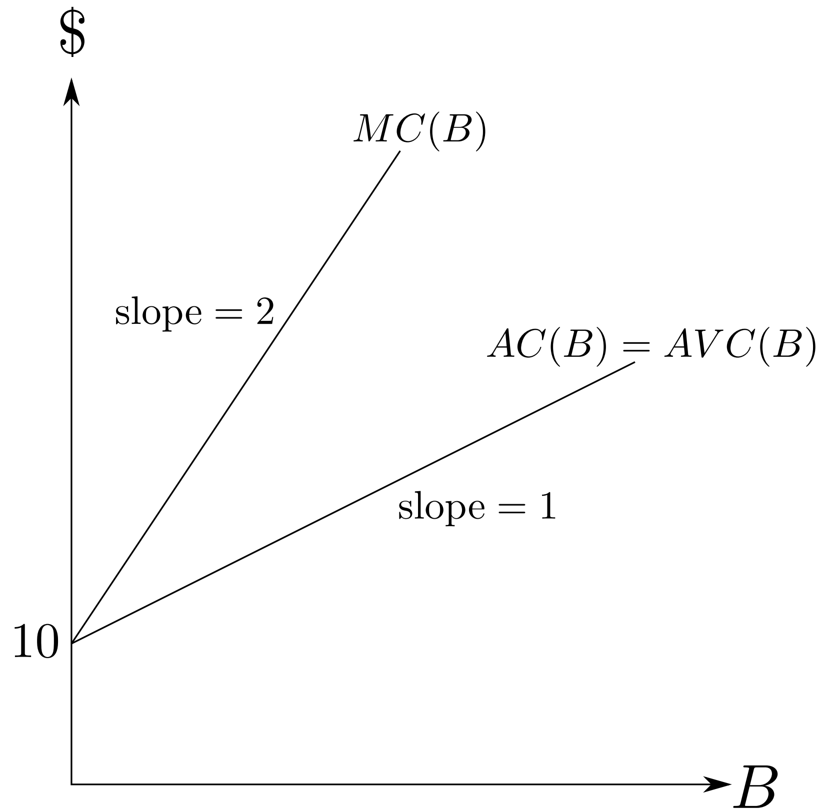
First, note that there are no fixed costs. Every term in the cost function varies with B so all of the costs are variable costs and average total costs will be the same as average variable costs. To find these average costs, we simply divide total costs by quantity:

$$AC(B) = \frac{C(B)}{B}$$

$$AC(B) = \frac{B^2 + 10B}{B}$$

$$AC(B) = B + 10$$

As argued above, $AVC(B)$ will also be equal to $B + 10$. Graphs of the cost curves are shown below.



(b) What is the price at which the firm would shut down?

The firm will shut down if it cannot cover its variable costs. This will occur when the price drops below the point where marginal cost intersects average variable cost:

$$MC(B_{sd}) = AVC(B_{sd})$$
$$2B_{sd} + 10 = B_{sd} + 10$$
$$2B_{sd} = B_{sd}$$

The only quantity that will solve this equation is zero. So at the shutdown price, the printer will produce zero books. Plugging this quantity back into either the marginal cost or average variable cost function will give us the shutdown price:

$$p_{sd} = MC(B_{sd})$$
$$p_{sd} = 2 \cdot 0 + 10$$
$$p_{sd} = 10$$

So the firm will not produce at any prices below \$10.

(c) Suppose that the market price is \$100. Determine the quantity of books the firm will decide to produce.

First note that \$100 is above the shutdown price, so the firm will produce a positive quantity of books. Given that the firm will produce, the optimal number of books to produce will be where price is equal to marginal cost. The books up to this point generate revenue that exceeds their costs ($p > MC$). The books after this point would generate more in costs than in revenues ($p < MC$). Setting price equal to marginal cost gives us:

$$p = MC(B)$$
$$p = 2B + 10$$
$$B = \frac{1}{2}p - 5$$

Plugging in \$100 for p gives us:

$$B = \frac{1}{2} \cdot 100 - 5$$
$$B = 45$$

(d) Find an expression giving the firm's profits as a function of the market price ($\pi(p)$). Your expression should only contain price and numerical constants.

Let's start by writing out profits in terms of price and the quantity of books produced:

$$\pi(p, B) = p \cdot B - C(B)$$
$$\pi(p, B) = p \cdot B - B^2 - 10B$$

We want this to be in terms of p only, so we need to replace B with an expression of p. We already found this expression in the previous part when we set price equal to marginal cost to find the optimal quantity. Plugging in the result for B in terms of p into the profit equation gives us:

$$\pi(p) = p \cdot (\frac{1}{2}p - 5) - (\frac{1}{2}p - 5)^2 - 10(\frac{1}{2}p - 5)$$
$$\pi(p) = \frac{1}{2}p^2 - 5p - \frac{1}{4}p^2 + 5p - 25 - 5p + 50$$
$$\pi(p) = \frac{1}{4}p^2 - 5p + 25$$

Notice that if we plug in our shutdown price of \$10, this expression gives us profits equal to zero. This should make sense, the shutdown price should be where profits go from positive to negative (in other words, where profits hit zero) when there are no fixed costs.

This expression covers prices above the shutdown price of \$10. Below the shutdown price, profits will be zero since there are no fixed costs that need to be paid. Therefore our final complete profit function is:

$$\pi(p) = \begin{cases} 0 & \text{if } p < 10 \\ \frac{1}{4}p^2 - 5p + 25 & \text{if } p \geq 10 \end{cases}$$

4. (20 points) A mattress manufacturer has three different factories. Two of the factories, factory A and factory B, are older and have the following costs functions:

$$C_A(M_A) = 10M_A^2 \tag{3}$$

$$C_B(M_B) = 10M_B^2 \tag{4}$$

where $C_A(M_A)$ is the total cost of producing M_A mattresses at factory A and $C_B(M_B)$ is the total cost of producing M_B mattresses at factory B. The third factory, factory C, uses a different technology and has constant marginal costs of \$100 per mattress, leading to the following total cost function:

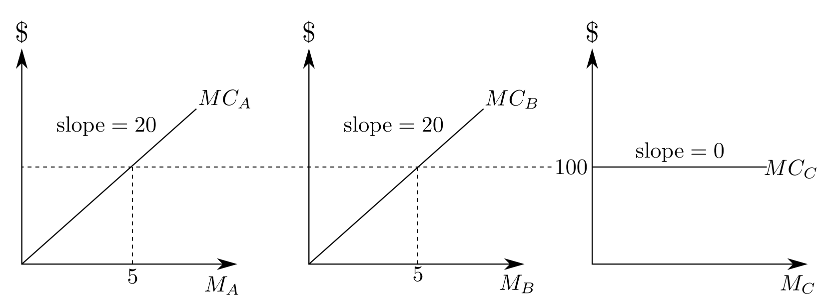
$$C_C(M_C) = 100M_C \tag{5}$$

(a) Graph the marginal cost curves for all three factories, using a separate graph for each.

First we need to get the equations for the marginal cost curves:

$$MC_A(M_A) = \frac{dC_A(M_A)}{dM_A}$$
$$MC_A(M_A) = 2 \cdot 10M_A$$
$$MC_A(M_A) = 20M_A$$
$$MC_B(M_B) = \frac{dC_B(M_B)}{dM_B}$$
$$MC_B(M_B) = 2 \cdot 10M_B$$
$$MC_B(M_B) = 20M_B$$
$$MC_C(M_C) = \frac{dC_C(M_C)}{dM_C}$$
$$MC_C(M_C) = 100$$

The graphs of these curves are shown below.



(b) Suppose that the mattress manufacturer wants to produce 20 mattresses. How many mattresses will the manufacturer produce at each factory? Be certain to show work to justify your answer.

The mattress manufacturer will want to distribute production across the factories to minimize total costs. This will be accomplished by distributing production such that marginal costs are equal across the factories. If marginal costs weren't equal across factories, production could be shifted from a high marginal cost factory to a low marginal cost factory to save money. Setting the marginal costs equal across factories gives us:

$$MC_A(M_A) = MC_B(M_B) = MC_C(M_C)$$

$$20M_A = 20M_B = 100$$

$$M_A = M_B = 5$$

So five mattresses will be produced at factory A and five mattresses will be produced at factory B. To find how many mattresses are produced at factory C, we can use the fact that the production at all three factories has to add up to 20 mattresses:

$$M_{total} = M_A + M_B + M_C$$

$$20 = 5 + 5 + M_C$$

$$M_C = 10$$

(c) Find the total costs of the manufacturer for producing M mattresses ($C(M)$) assuming the manufacturer optimally distributes production across its factories.

Notice from the previous part that the manufacturer will typically produce five mattresses at factory A, five at factory B and then the remainder at factory C. This is because factory C has constant marginal costs of \$100 per mattresses. Marginal costs would get above this at factory A or factory B if more than five mattresses were produced at either one. So in general, if we want to produce M mattresses, we will do so by producing five at factory A, five at factory B and $M - 10$ at factory C leading to the following total costs:

$$C(M) = C_A(M_A) + C_B(M_B) + C_C(M_C)$$

$$C(M) = C_A(5) + C_B(5) + C_C(M - 10)$$

$$C(M) = 10 \cdot 5^2 + 10 \cdot 5^2 + 100 \cdot (M - 10)$$

$$C(M) = 100M - 500$$

However, this only works if we want to produce at least 10 mattresses. If we are producing fewer than 10 mattresses, we won't use factory C since the marginal costs at factory C would be higher than the marginal costs at either factory A or B if we are splitting output between them (you can see this by looking at the marginal costs curves graphed above). If we aren't using factory C, the optimal distribution of production will be based on where marginal costs at factory A equal marginal costs at factory B and M_A and M_B add up to our desired M_{total}. Starting with setting our marginal costs equal, we get:

$$MC_A(M_A) = MC_B(M_B)$$

$$20M_A = 20M_B$$
$$M_A = M_B$$

Now we can plug this result into our expression relating M_A and M_B to total output:

$$M_A + M_B = M_{total}$$
$$M_A + M_A = M_{total}$$
$$2M_A = M_{total}$$
$$M_A = \frac{1}{2}M_{total}$$

So when we want to produce fewer than ten mattresses, M_A will be one half of the total output, M_B will be one half of the total output and M_C will be zero. This will give us total costs of:

$$C(M) = C_A(M_A) + C_B(M_B) + C_C(M_C)$$
$$C(M) = C_A(\frac{1}{2}M) + C_B(\frac{1}{2}M) + C_C(0)$$
$$C(M) = 10 \cdot (\frac{1}{2}M)^2 + 10 \cdot (\frac{1}{2}M)^2 + 0$$
$$C(M) = 5M^2$$

So our final cost function for the mattress manufacturer is:

$$C(M) = \begin{cases} 5M^2 & \text{if } M < 10 \\ 100M - 500 & \text{if } M \geq 10 \end{cases}$$

Notice that the two segments of this cost function match up at that critical quantity of 10 mattresses where we start to use the third factory ($C(10) = 500$ when using either segment's equation).