# Midterm 2 - Solutions

You have until 4:50pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that individuals can consume fractions of units and firms can use fractions of units of inputs and produce fractions of units of output. Remember to put your name on the exam. Good luck!

**Name:**

1. (20 points) Abel's demand for bread ($B$) and water ($W$) in terms of the price of bread ($p_B$), the price of water ($p_W$) and his income ($I$) are given by the following demand equations:

$$B(p_B, p_W, I) = \frac{I}{3p_B} \tag{1}$$

$$W(p_B, p_W, I) = \frac{2I}{3p_W} \tag{2}$$

(a) Find an expression for the price elasticity of demand for bread. Your answer should contain only constants, price and income.

The price elasticity of demand is given by:

$$\eta = \frac{p_B}{B(p_B)} \frac{dB(p_B)}{dp_B}$$

We want this to be a function of only prices and income, so we need to replace $B(p_B)$ with the appropriate function in terms of $p_B$:

$$\eta = \frac{p_B}{\frac{I}{3p_B}} \frac{dB(p_B)}{dp_B}$$

$$\eta = \frac{3p_B^2}{I} \frac{dB(p_B)}{dp_B}$$

Now we need to evaluate the derivative to put get the second part of the elasticity expression in terms of income and prices:

$$\eta = \frac{3p_B^2}{I} \cdot (-1) \frac{I}{3p_B^2}$$

$$\eta = -1$$

(b) Suppose that bread is currently being sold for \$5 a loaf, water is being sold for \$1 a bottle and Abel has \$120 in income. Use your expression from part (a) to determine whether the bakery's revenue will increase, decrease or stay the same if it decides to increase the price of a loaf of bread by a small amount.

From part (a), we know that demand for bread is unit elastic. Therefore, when price is increased by one percent, demand decreases by exactly one percent, perfectly offsetting any revenue gains. So a change in price, whether it is an increase or a decrease in price will lead to no change in revenue. To see this without using the elasticity from part (a), you can write out revenue as a function of $p_B$:

$$R(p_B) = p_B \cdot B(p_B)$$

$$R(p_B) = p_B \cdot \frac{I}{3p_B}$$

$$R(p_B) = \frac{I}{3}$$

From this expression, we can see that revenue does not depend on the price of bread. This is because we have the unique case of a demand curve that is unit elastic along the entire curve.

(c) The price of water increases to \$2 while the price of bread and income stay the same as in part (b). Decompose the change in demand for bread from this price change into the component due to the substitution effect and the component due to the income effect.

First let's find the optimal quantities of bread and water before and after the price change. To get the optimal bundle before the price change, we need to plug in the original prices and income into the demand functions:

$$B_o = B(5, 1, 120) = \frac{120}{3 \cdot 5}$$

$$B_o = 8$$

$$W_o = W(5, 1, 120) = \frac{2 \cdot 120}{3 \cdot 1}$$

$$W_o = 80$$

Next we can find the new optimal bundle by switching the price of water to \$2 in these demand functions:

$$B_f = B(5, 2, 120) = \frac{120}{3 \cdot 5}$$

$$B_f = 8$$

$$W_f = W(5, 2, 120) = \frac{2 \cdot 120}{3 \cdot 2}$$

$$W_f = 40$$

So overall, demand for bread has not changed. However, this does not mean that the changes in demand for bread due to the income effect and due to the substitution effect are zero. It just means that they will cancel each other out. To get the changes in demand due to the income and substitution effect, we need to find the optimal bundle under the new prices but at an income level adjusted to keep the original bundle, $(B_o, W_o)$, just affordable under the new prices. This income is:

$$\tilde{I} = p_B B_o + p'_W W_o$$
$$\tilde{I} = 5 \cdot 8 + 2 \cdot 80$$
$$\tilde{I} = 200$$

Plugging in this adjusted income and the prices after the price change in the demand function for bread will give us the quantity of bread in the intermediate bundle:

$$B_i = B(5, 2, 200) = \frac{200}{3 \cdot 5}$$
$$B_i = \frac{40}{3}$$

Now we can find the change in bread due to the substitution effect and due to the income effect by comparing the original, intermediate and final bundles. The change in bread due to the substitution effect is the difference between the original and intermediate bundles (this is the difference driven by the change in prices holding effective income constant). The change in bread due to the income effict is the difference between the intermediate and final bundles (this is the difference driven by the change in effictive income holding prices constant).

$$\Delta B_{sub} = B_i - B_o = \frac{40}{3} - 8 = \frac{16}{3}$$
$$\Delta B_{inc} = B_f - B_i = 8 - \frac{40}{3} = -\frac{16}{3}$$

2. (30 points) The demand for corn is given by the following linear demand function:

$$D(p) = 100 - 20p \tag{3}$$

where $D(p)$ is the number of bushels of corn demanded at a price of $p$ per bushel. The supply of corn is given by:

$$S(p) = 5p \tag{4}$$

where $D(p)$ is the number of bushels of corn supplied by farmers at a price of $p$ per bushel.

(a) What is the equilibrium price of a bushel of corn? What is the equilibrium quantity of bushels sold?

Setting supply equal to demand will give us the equilibrium price of a bushel of corn:

$$D(p) = S(p)$$

$$100 - 20p = 5p$$

$$25p = 100$$

$$p = 4$$

To get the equilibrium quantity, we can plug this price into either the demand function or the supply function:

$$D(4) = 100 - 20 \cdot 4 = 20$$

$$S(4) = 5 \cdot 4 = 20$$

(b) Suppose that the government decides to subsidize corn growers by giving them a subsidy of $5 per bushel of corn. So if consumers pay a price of $p$ per bushel, the corn growers recieve a price of $p$+5 per bushel. What will the new equilibrium price paid by consumers be and what will the new equilibrium quantity of corn be once the subsidy is in place?

Note that this subsidy works just like a quantity tax, only the tax is negative $5. Finding equilibrium follows the same logic we used for a quantity tax. At the equilibrium prices, the quantity demanded by the consumers at the price they pay should be equal to the quantity farmers are willing to supply at the price they actually get to keep. If consumers are paying a price of $p$, farmers get to keep an amount equal to $p + 5$ (the price paid by consumers plus the subsidy). So the following equation will give us the equilibrium price paid by consumers:

$$D(p) = S(p + 5)$$

$$100 - 20p = 5 \cdot (p + 5)$$

$$100 - 20p = 5p + 25$$

$$25p = 75$$

$$p = 3$$

So consumers will now pay a lower price than before the subsidy was put in place. The equilibrium quantity can be found by plugging this new price into the demand function:

$$D(3) = 100 - 20 \cdot 3 = 40$$

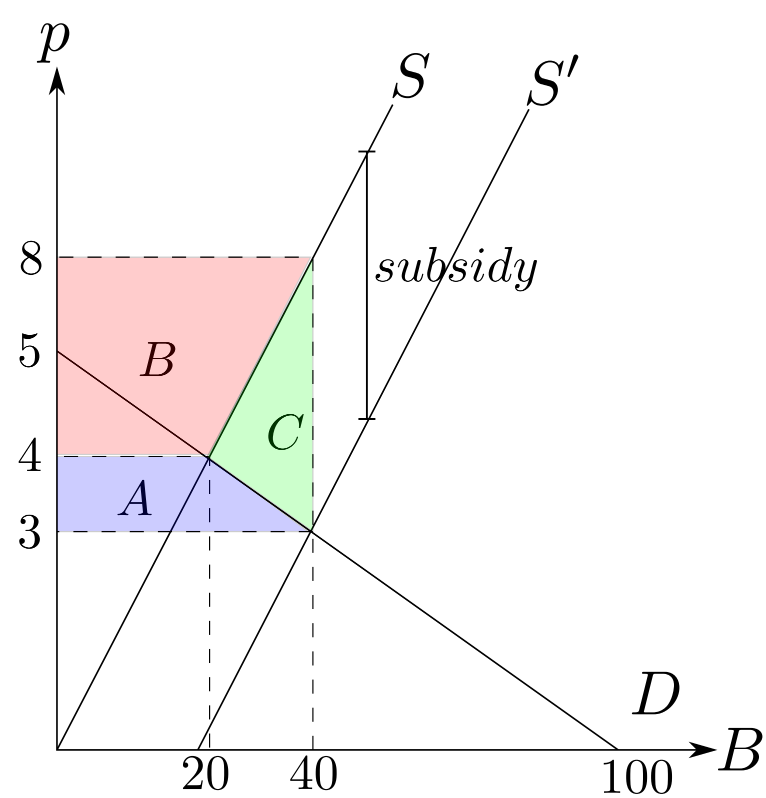
Note that if you wanted to find the equilibrium quantity using the supply function, you would plug in the price paid by consumers plus the subsidy.

(c) How much will the government have to spend in total on subsidies?

From the previous part, we know that the new equilibrium quantity is 40 bushels. At \$5 in subsidy per bushel, the total amount spent in subsidies will be $5 \cdot 40$ or \$200.

(d) On a graph, show the deadweight loss generated by the subsidy program. Label all relevant points with their numerical values. Also calculate the numerical value of this deadweight loss.

Graphically, we can think of the subsidy shifting the supply curve down by \$5 since firms are now willing to supply the same amount they once supplied at a price of $p$ at a price of $p - 5$ (the subsidy will bring the firm's net price back up to $p$). This will lead to a new higher equilibrium quantity, a lower price paid by consumers and a higher price received by the farmers (once the subsidy is added in). This is shown on the graph below:



On the graph, the total amount paid in subsidies is the area $A + B + C$ (the height of this rectangle is the size of the subsidy per unit and the width is the number of units the government must pay a subsidy on). Consumer surplus has increased by the amount $A$ and producer surplus has increased by the amount $B$. So most of the loss in total surplus from the subsidy is being transferred into a gain in consumer and producer surplus (the opposite of what happened with a tax where consumer and producer surplus was transferred to tax revenue). However, area $C$ is lost surplus that does not go into either consumer surplus or producer surplus. This is the deadweight loss associated with the subsidy program. The area of that triangle is:

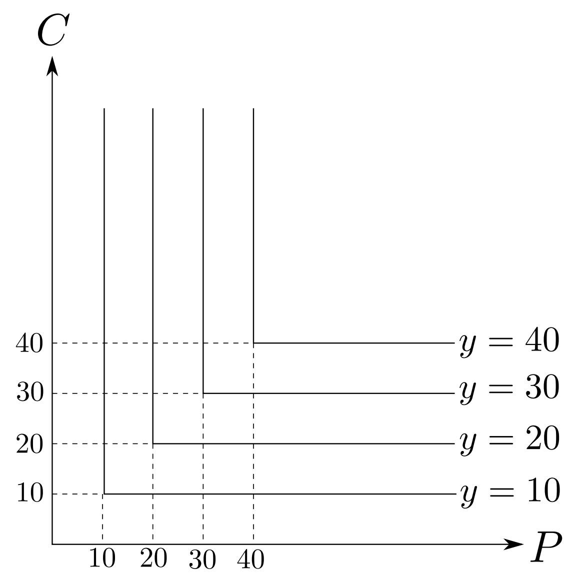
$$DWL = \frac{1}{2}(B_{\text{w/ subsidy}} - B_{\text{w/o subsidy}}) \cdot \text{Subsidy}$$

$$DWL = \frac{1}{2}(40 - 20) \cdot 5$$
$$DWL = 50$$

3. (25 points) For each technology described below, draw a set of four isoquants corresponding to 10, 20, 30 and 40 units of output. Where possible, give exact numerical values for slopes, kinks and intercepts.

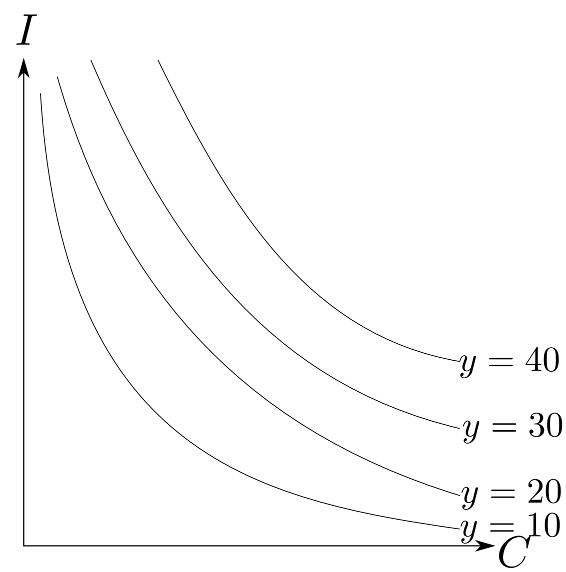
   (a) A firm uses programmers ($P$) and computers ($C$) to produce video games. A programmer with one computer can produce one video game. Extra computers do not increase the productivity of a programmer who already has a computer. No more than one programmer can use each computer.

   This is the description of a fixed proportions technology, each unit of output requires exactly one computer and one programmer. If we start with an equal number of computers and programmers, increasing one input without increasing the other will not allow us to produce any additional output. This gives us L-shaped isoquants, with the corners of these isoquants at the points where the number of programmers exactly equals the number of computers.



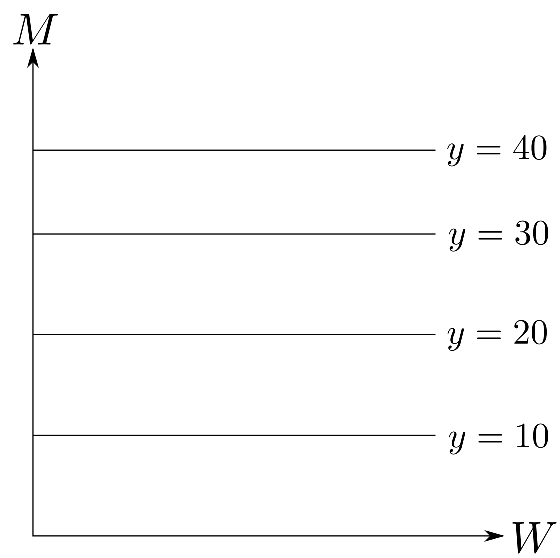

   (b) A restaurant uses ingredients ($I$) and cooks ($C$) to produce entrees. Each additional pound of ingredients increases the number of entrees produced by exactly as much as the previous pound of ingredients, regardless of the current combination of ingredients and cooks. Each additional cook increases the number of entrees produced but by a smaller amount than the previous cook.

   From the description, we know that both inputs always have positive marginal products. This tells us that we will have downward sloping isoquants (if we take away some of one input, we will have to add more of the other to keep output the same). On a graph with cooks on the horizontal axis and ingredients on the vertical axis, the slope of an isoquant will be given by $-\frac{MP_C}{MP_I}$. As we move down and right along an isoquant, the numerator of this expression will decrease (the marginal product of cooks is diminishing). The denominator will stay constant (the marginal product of ingredients is the same regardless of the current amount of ingredients being used). So overall, the magnitude of the slope will be getting smaller telling us that the isoquants get flatter as we move down and to the right.



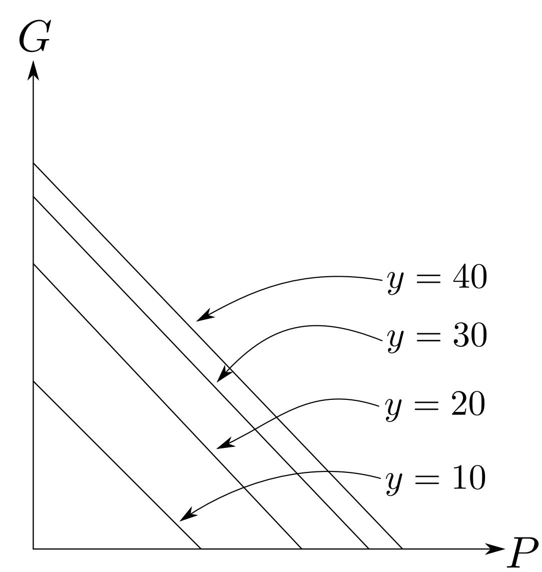

(c) A firm uses machines ($M$) and workers ($W$). With the latest machine technology, it turns out that workers are now obsolete. Changing the number of workers has no impact on total output as the machines can now handle every aspect of production. Increasing the number of machines always increases the amount of total output.

The description is telling us that the marginal product of workers is zero. On a graph with workers on the horizontal axis and machines on the vertical axis, this will give us horizontal lines (moving to the right by adding more workers will have no effect on output).




(d) A firm uses gears ($G$) and pulleys ($P$) to move cargo (you can think of output as the total tons of cargo moved). The technical rate of substitution is constant and the firm's technology exhibits increasing returns to scale.

A constant technical rate of substitution tells us that we have linear isoquants. If the technology exhibits increasing returns to scale, these isoquants should get more closely spaced together as output levels get higher (the firm is requiring smaller and smaller increases in inputs to achieve each increase in output).



4. (25 points) There are two types of consumers of hamburgers in Williamsburg, college students ($C$) and retirees ($R$). There are 50 students and 100 retirees. The demand for a single college student is given by:

$$H_C(p) = 100 - 2p \tag{5}$$

where $H_C(p)$ is the number of hamburgers a college student demands at a price of $p$. The demand for a single retiree is given by:

$$H_R(p) = 200 - 2p \tag{6}$$

(a) Is demand for hamburgers more elastic for a college student or a retiree? You only need to consider prices at which both types of consumers have positive demand. Be certain to fully justify your answer.

Let's begin by deriving expressions for the elasticity of each consumer type. First, the college student:

$$\eta_C = \frac{p}{H_C(p)} \frac{dH_C(p)}{dp}$$

$$\eta_C = \frac{p}{100 - 2p} \cdot (-2)$$

$$\eta_C = \frac{-2p}{100 - 2p}$$

Next, let's do the same for the retiree:

$$\eta_R = \frac{p}{H_R(p)} \frac{dH_R(p)}{dp}$$

$$\eta_R = \frac{p}{200 - 2p} \cdot (-2)$$

$$\eta_R = \frac{-2p}{200 - 2p}$$

Notice that the numerators of these two expressions are identical. For any given price, the denominator for $\eta_C$ will be smaller than the denominator for $\eta_R$, making $\eta_C$ larger in magnitude at any given price than $\eta_R$. So a college student has more elastic demand than a retiree at any price where both have positive demand.

(b) Would you expect a college student's demand for fast food in general (which includes hamburgers) to be more or less elastic than a college student's demand for hamburgers? Be certain to explain your answer.

We would typically expect demand for fast food in general to be less elastic than demand for hamburgers. This is because of the number of available close substitutes. If the price of hamburgers and only hamburgers rises, we would expect college students to respond by switching to close substitutes like hot dogs or pizza. This would imply that demand for hamburgers is very responsive to a change in price, in other words very elastic. However, if the price of all fast food rises, there aren't many good substitutes so we would not expect to see demand fall off nearly as much, implying less elastic demand.

(c) Suppose that Williamsburg needs to raise \$1000 in tax revenue and is planning to do so with a value tax placed on all fast food. Would this tax generate more or less deadweight loss than a value tax placed on just hamburgers (assuming that this tax on hamburgers would also generate \$1000 in tax revenues)? Be certain to justify your answer.

Following the logic of the previous answer, we would expect the demand for hamburgers to be more elastic than the demand for fast food in general. Therefore a tax on hamburgers will generate a bigger decline in quantity for hamburgers than a similar tax on fast food would. This greater decline in quantity would generate more deadweight loss (the deadweight loss is the consumer and producer surplus that is no longer realized because those units are no longer being consumed).

(d) Derive an expression for the market demand curve for hamburgers in Williamsburg. On a graph with hamburgers on the horizontal axis and price on the vertical axis, graph this market demand curve labelling all intercepts, slopes and kinks with their numerical values.

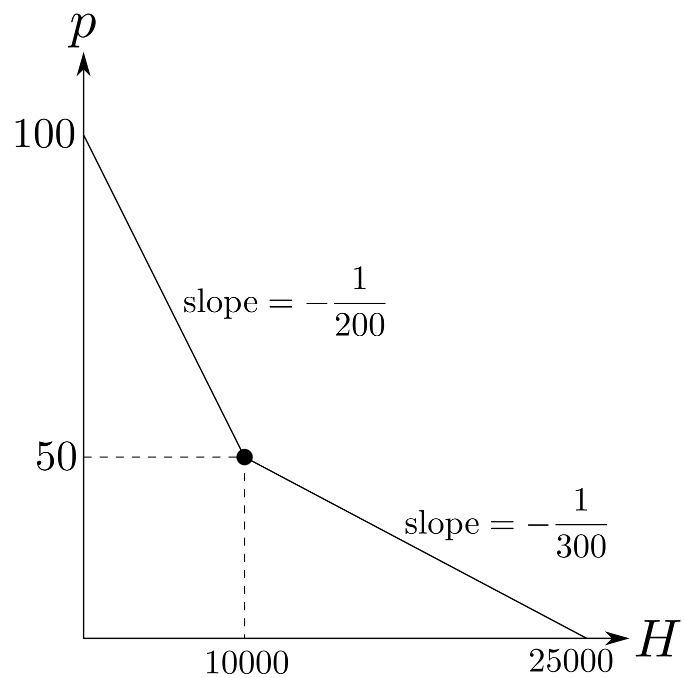
First, note that college student demand for hamburgers goes to zero when the price goes to \$50. For retirees, their demand hits zero at a price of \$100. So we have three distinct price ranges to consider for our market demand curve: prices above \$100, prices between \$100 and \$50, and prices below \$50. At prices above \$100, no consumers will buy hamburgers. At prices between \$50 and \$100, the 100 retirees will be demanding positive quantities of hamburgers but college students will still not be buying hamburgers. When the price drops below \$50 everyone, the 50 students and 100 retirees, will be buying positive quantities. Therefore, our market demand curve will be given by:

$$H(p) = 0 \text{ for } p > 100$$

$$H(p) = 100H_R(p) = 100 \cdot (200 - 2p) = 20000 - 200p \text{ for } 50 < p \leq 100$$

$$H(p) = 100H_R(p) + 50H_C(p) = 100 \cdot (200 - 2p) + 50 \cdot (100 - 2p) = 25000 - 300p \text{ for } p \leq 50$$

This market demand curve is shown on the graph below:




(e) Assuming the supply curve for hamburgers is linear and upward sloping, show the effects of a value tax of 20% placed on hamburgers on the price paid by consumers and the price received by producers on your graph from part (c). The tax is levied on consumers, meaning they are the ones who must pay the tax to the government. Note that you will not be able to give exact numbers for the prices but you should give exact numerical values for the slopes and intercepts related to the demand curve.

Note that what consumers were willing to buy at price $p$ before, they will now only be willing to buy at a price of $\tilde{p}$ where $p = (1+\tau)\tilde{p}$ because they will also have to pay the tax to the government. So the new intercept for the market demand curve (in terms of the price consumers are willing to pay producers, $\tilde{p}$) will be $\frac{200}{1.2}$ rather than 200 and the kink will occur at a price of $\frac{100}{1.2}$ rather than 100. To think of how the slopes change, note the following:

$$\text{slope} = \frac{\Delta \tilde{p}}{\Delta H} = \frac{\frac{1}{1+\tau}\Delta p}{\Delta H}$$

$$\text{slope} = \frac{1}{1.2}\frac{\Delta p}{\Delta H}$$

So the new slopes are simply the original segment slopes divided by 1.2. This new demand curve is shown on the graph below along with a linear supply curve. To see the effects of the tax on prices, we can compare the original price $p_o$ (the intersection of the original demand curve and the supply curve) to the new price received by producers $p_p$ (the intersection of the rotated demand curve and the supply curve) and the new price paid by consumers (the price given by the original demand curve at the new equilibrium quantity where the rotated demand curve intersects the supply curve). As you can see from the graph, the tax is being split between the consumer and producer. The producer's price has fallen below the original price while the consumer's price is now above the original price. Note that you could have also had the supply curve intersecting the lower segment of the market demand curve.