Midterm 2 - Solutions

You have until 12:20pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Non-graphing calculators may be used (no graphing calculators or phones can be used). You may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can produce fractions of units and charge non-integer prices (so a firm could produce 82.4 units and sell at a price of $5.325 per unit). Remember to put your name on the exam. Good luck!

Name: **ID Number:**

1. (10 points) It costs a truck manufacturer $100,000 to set up a factory. Once the factory is set up, each additional truck built by the manufacturer adds $5,000 to the manufacturer's total costs.

   (a) Write down expressions for the manufacturer's average total costs, average fixed costs, average variable costs and marginal costs as functions of the number of trucks produced, T.

   Notice that the $100,000 to set up the factory does not depend on the number of trucks produced. Therefore, this $100,000 should be though of as fixed costs. The marginal costs are equal to $5,000 for every truck, so total costs will increase linearly with T at a rate of $5,000 per truck. This tells us that the variable costs will be $5000 \cdot T$. So our cost functions are the following:

$$C(T) = FC + VC(T)$$

$$C(T) = 100000 + 5000T$$

$$AC(T) = \frac{C(T)}{T} = \frac{100000}{T} + 5000$$

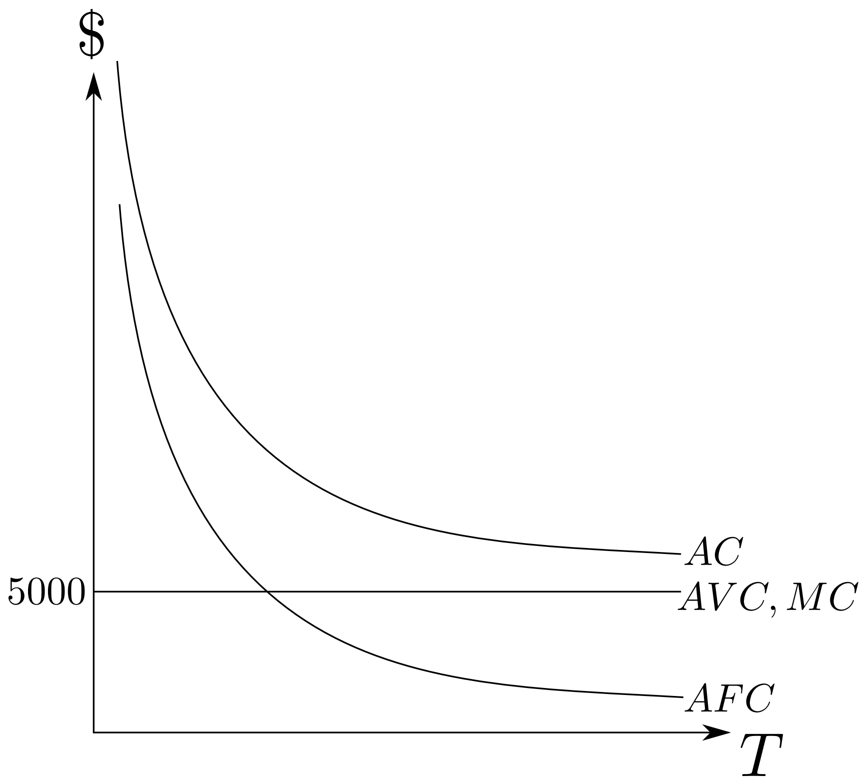
$$AFC(T) = \frac{FC}{T} = \frac{100000}{T}$$

$$AVC(T) = \frac{VC(T)}{T} = 5000$$

$$MC(T) = \frac{dC(T)}{dT} = 5000$$

   (b) Graph the average total cost, average variable cost and marginal cost curves on a graph with trucks on the horizontal axis and price on the vertical axis.

   This graph of cost curves is actually a bit simpler than our typical graph because of the constant marginal costs. Both the marginal cost curve and the average variable cost curve will be a horizontal line at a value of $5,000. The average fixed cost curve will be the standard average fixed cost curve: a negatively sloped curve that approached infinity as T approaches zero and zero as T approaches infinity. The average total cost curve is the same as this average fixed cost curve only shifted up by the average variable costs of \$5,000. So instead of approaching the horizontal axis as T gets large, it approaches \$5,000.



2. (30 points) A farm produces vegetables using water and fertilizer. The pounds of vegetables produced, V, is given by the following function:

$$f(W, F) = W^2F^2 \tag{1}$$

where W is the number of gallons of water used and F is the number of pounds of fertilizer used. Given this production function, the marginal product of water and the marginal product of fertilizer are the following:

$$MP_W = 2WF^2 \tag{2}$$

$$MP_F = 2W^2F \tag{3}$$

The price of a gallon of water is \$1 and the price of a pound of fertilizer is \$16. The price of a pound of vegetables is \$20.

(a) Determine which of the following properties hold for this production function: diminishing marginal product of each input, diminishing technical rate of substitution, monotonicity, convexity. For each, explain in one sentence why it does or does not hold.

- Marginal product of water is increasing rather than diminishing. This is easily seen by the fact that MP_W is larger when W is larger ($2WF^2$ increases as W increases).
- Marginal prodcut fertlizier is also increasing because MP_F is larger when F is larger ($2W^2F$ increases when F increases).
- The technical rate of substitution is diminishing. On a graph with F on the horizontal axis and W on the vertical axis, the technical rate of substitution would be:

$$TRS = -\frac{MP_F}{MP_W}$$

$$TRS = -\frac{2W^2F}{2WF^2}$$

$$TRS = -\frac{W}{F}$$

As you move from left to right along an isoquant, increasing F and decreasing W, the magnitidue of the TRS is getting smaller since the numerator is decreasing and the denominator is increasing, giving us a diminishing technical rate of substitution.
- The production function is monotonic. From the marginal product functions we can see that both marginal products are always greater than or equal to zero, so increasing either input will always lead to a level of output greater than or equal to the original level of ouput.
- The production function is convex. We can see this because the technical rate of substitution is diminishing giving us convex-shaped isoquants. If the isoquants have a convex shape, a line connecting any two points on the isoquant would lie entirely above the isoquant telling us that an average of the two combinations of inputs (meaning a point on that line) would produce more output than either original combination of inputs.

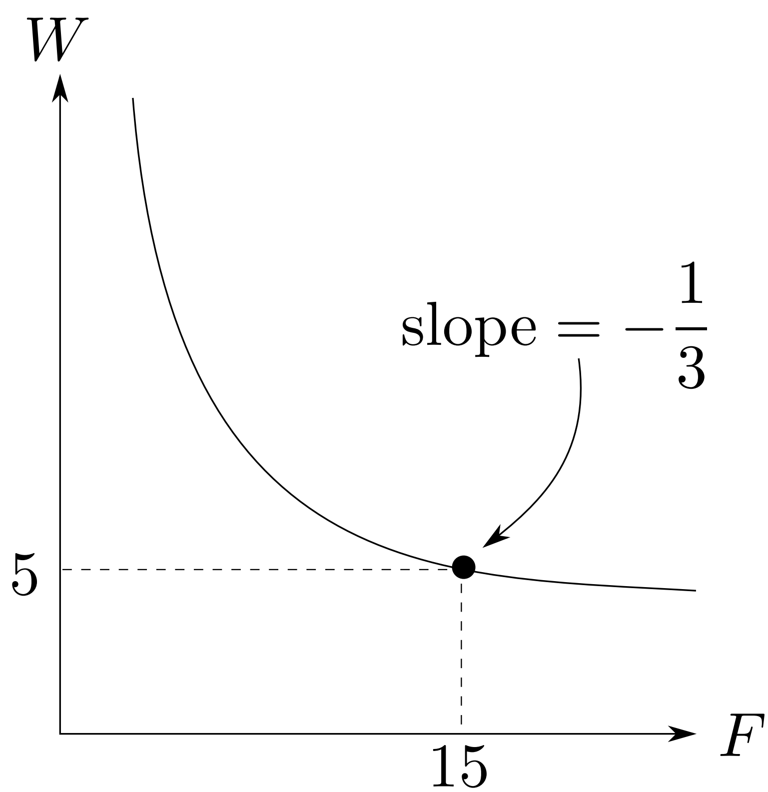
(b) On a graph with with fertilizer on the horizontal axis and water on the vertical axis, draw the isoquant passing through the point (5 gallons of water, 15 pounds of fertilizer) and label the slope of the isoquant at this point with its numerical value.

As discussed above, the production function exhibits a diminishing technical rate of substitution so the isoquant will be a convex curve passing through the point $(15, 5)$. The slope of the isoquant at that point is simply the value of the technical rate of substitution plugging in the appropriate values for F and W:

$$TRS = -\frac{W}{F}$$

$$TRS = -\frac{5}{15}$$

$$TRS = -\frac{1}{3}$$



(c) Find an expression for the optimal amount of fertilizer as a function of vegetables, $F(V)$, and the optimal amound of water as a function of vegetables, $W(V)$.

At the optimal combination of fertlizer and water, the ratio of the prices of the two inputs will be equal to the ratio of the marginal products of the two inputs. If this weren't the case, it would be possible to use more of the input with a larger marginal product to price ratio and less of the input with the smaller marginal product to prict ratio to lower costs while keeping output the same. So, setting the ratio of the prices equal to the ratio of the marginal products, we get:

$$\frac{p_F}{p_W} = \frac{MP_F}{MP_W}$$

$$\frac{p_F}{p_W} = \frac{2W^2F}{2WF^2}$$

$$\frac{p_F}{p_W} = \frac{W}{F}$$

$$W = \frac{p_F}{p_W}F$$

Now we can plug this result into our production function to find F as a function of V:

$$V = f(F, W)$$

$$V = W^2F^2$$

$$V = \left(\frac{p_F}{p_W}F\right)^2 F^2$$

$$V = \left(\frac{p_F}{p_W}\right)^2 F^4$$

$$F^4 = \left(\frac{p_W}{p_F}\right)^2 V$$

$$F = \left(\frac{p_W}{p_F}\right)^{\frac{2}{4}} V^{\frac{1}{4}}$$

$$F = \left(\frac{p_W}{p_F}\right)^{\frac{1}{2}} V^{\frac{1}{4}}$$

To get W as a function of V, we can plug this result back into our equation relating W to F:

$$W = \frac{p_F}{p_W}F$$

$$W = \frac{p_F}{p_W}\left(\frac{p_W}{p_F}\right)^{\frac{1}{2}} V^{\frac{1}{4}}$$

$$W = \left(\frac{p_F}{p_W}\right)^{\frac{1}{2}} V^{\frac{1}{4}}$$

Plugging in the prices given in the problem will give us our final expressions for $F(V)$ and $W(V)$:

$$F(V) = \left(\frac{1}{16}\right)^{\frac{1}{2}} V^{\frac{1}{4}}$$

$$F(V) = \frac{1}{4} V^{\frac{1}{4}}$$

$$W(V) = \left(\frac{16}{1}\right)^{\frac{1}{2}} V^{\frac{1}{4}}$$

$$W(V) = 4V^{\frac{1}{4}}$$

(d) What is the lowest possible cost of producing 256 pounds of vegetables?

Our functions from the previous part give us the lowest cost combination of fertilizer and water for any given quantity of vegetables. Plugging in 256 for V in these equations gives us:

$$F(256) = \frac{1}{4} \cdot 256^{\frac{1}{4}}$$

$$F(256) = 1$$

$$W(256) = 4 \cdot 256^{\frac{1}{4}}$$

$$W(256) = 16$$

So our total costs of producing 256 pounds of vegetables will be:

$$C(256) = p_W \cdot W(256) + p_F \cdot F(256)$$

$$C(256) = 1 \cdot 16 + 16 \cdot 1$$

$$C(256) = 32$$

3. (25 points) A window factory makes windows (W) out of frames (F) and panes of glass (G). Each window requires one frame and four panes of glass. The cost of a frame is \$10. The cost of each pane of glass is \$5. The factory can sell windows for \$50 each.

   (a) Write down a production function $f(F, G)$ that gives the number of windows the factory can produce as a function of number of frames and panes of glass it uses.

   This is an example of a fixed proportions technology, frames and panes of glass must always be combined in a fixed ratio. Having an extra of one of the inputs but not of the other will not increase output. This type of production technology will be captured by a min function. Note that the number of windows produced is constrained by either the number of frames available or one quarter of the number of panes of glass available, whichever is smaller. So our production function is:

   $$f(F, G) = \min(F, \frac{1}{4}G)$$

   Note that unlike when we did perfect complements with consumers in order to be correct you need the proper magnitudes for the coefficients, not just the ratio of the magnitudes. This is because our result from the function should be the number of windows produced. For example, the function $f(F, G) = \min(4F, G)$ still captures the ratio of frames to panes of glass but it would imply that one frame and four panes of glass produces four windows.

   (b) In the short run, the firm has 20 window frames but can order as much glass as it wants to use. What is the profit-maximizing amount of glass the firm should use in the short run, how many windows will they produce and what will the factory's total costs be?

   If F is fixed at 20 in the short run, the production fuction is reduced to:

   $$f(20, G) = \min(20, \frac{1}{4}G)$$

   So for any value of G less than 80, the number of windows we can produce will be $\frac{1}{4}G$. For any value of G greater than 80, we will be able to produce 20 windows. Clearly we don't want to pay for more than 80 panes of glass since any extra panes after 80 do not lead to any additional output. So we can restrict our attention to values of G between zero and 80. Let's start by considering a vlue of G less than 80. Increasing G by four would allow us to produce one more window increasing our revenues by \$50, the price of a window. Costs would go up by \$20, the cost of the additional four panes of glass. Notice that we don't have to worry about the cost of the frames, as we have to pay for 20 frames no matter what. So each time we increase G by four, revenues go up by more than costs, meaning the profits increase (every time G goes up by four, profits go up by \$30). Given that increasing G will always increase profits when G is less than 80, we will increase it all the way to 80. We won't go beyond 80 because increasing G beyond 80 adds to costs but not to revenues since we're constrained by the number of frames at that point. Total costs will be the money spent on frames, \$10 times the 20 frames, plus the money spent on glass, \$5 times the 80 panes of glass, giving us total costs of \$600.

(c) In the long run both frames and glass are variable inputs. Suppose that the firm wants to produce the same number of windows in the long run as the number it produced in part (b). Will the costs of producing this many windows in the long run be more expensive, less expensive or the same as the costs you found in part (b)? Be certain to justify your answer.

If we are going to produce the same quantity of windows, 20, in the long run, the costs will end up be exactly the same. Unlike typical cases where we could substitute away from an expensive input toward a cheaper input in the long run, there is no possibility of substituting frames from glass or the other way around. The only way that our costs would be reduced is if we had too much of one input and it was going unused. Because we oped to use all of the frames and just enough glass for the 20 frames, there were no excess inputs so our combination of inputs will be exactly the same in the long run as in the short run leading to the same costs.

4. (20 points) Suppose that there are two types of consumers of coffee. Each consumer of type A has the following demand for coffee:

$$C_A(p) = 20 - 2p \tag{4}$$

where $C_A(p)$ is the number of cups of coffee demanded when the price of a cup of coffee is p. Each consumer of the type B has the following demand funtion:

$$C_B(p) = 10 - \frac{1}{2}p \tag{5}$$

where $C_B(p)$ is the number of cups of coffee demanded when the price of a cup of coffee is p. There are 10 consumers of type A and 10 consumers of type B.

(a) Calculate the price elasticity of demand for each consumer type at a price of \$4.

Let's begin with consumer type A:

$$\varepsilon_A = \frac{p}{C_A(p)} \frac{dC_A(p)}{dp}$$

$$\varepsilon_A = \frac{p}{20 - 2p}(-2)$$

$$\varepsilon_A = \frac{4}{20 - 2 \cdot 4}(-2)$$

$$\varepsilon_A = -\frac{2}{3}$$

Now for consumer type B:

$$\varepsilon_B = \frac{p}{C_B(p)} \frac{dC_B(p)}{dp}$$

$$\varepsilon_B = \frac{p}{10 - \frac{1}{2}p}(-\frac{1}{2})$$

$$\varepsilon_B = \frac{4}{10 - \frac{1}{2} \cdot 4}(-\frac{1}{2})$$

$$\varepsilon_B = -\frac{1}{4}$$

(b) Would you expect the price elasticity of demand for all beverages (including coffee) to be larger or smaller in magnitude than the elasticities you found in part (a)? Be certain to explain your answer.

There are closer substitutes for coffee than there are for beverages as a whole. So if the price of coffee increases, many customers will switch from coffee to other beverages, leading to a large price elasticity of demand. If the price of beverages as a whole go up, there are very few reasonable substitutes for consumers to switch to, leading to a smaller change in quantity and a smaller magnitude for the price elasticity of demand.

(c) Derive an expression for the overall market demand as a function of price, $C(p)$, and graph the demand curve labeling any relevant slopes, intercepts and kinks with their numerical values.

First, notice that at prices above \$10, consumers of type A drop out of the market. At a price of \$20, consumers of type B drop out of the market. So we need to consider market demand over three price ranges: $0 \leq p < 10$, $10 \leq p < 20$ and $20 \leq p$. For prices greater than \$20, market demand is simply zero. For prices between \$20 and \$20, only consumers of type B are demanding positive quantities so market demand will be:

$$C(p) = 10 \cdot C_B(p)$$
$$C(p) = 10 \cdot (10 - \frac{1}{2}p)$$
$$C(p) = 100 - 5p$$

For prices below \$10, all consumers have positive demand so the market demand will be:

$$C(p) = 10 \cdot C_A(p) + 10 \cdot C_B(p)$$
$$C(p) = 10 \cdot (20 - 2p) + 10 \cdot (10 - \frac{1}{2}p)$$
$$C(p) = 300 - 25p$$

So the complete market demand function is:

$$C(p) = \begin{cases} 0 & \text{if } p > 20 \\ 100 - 5p & \text{if } 20 \geq p > 10 \\ 300 - 25p & \text{if } 10 \geq p > 0 \end{cases}$$

For graphing, it is simpler to switch to the inverse demand function:

$$p(C) = \begin{cases} 20 - \frac{1}{5}C & \text{if } 50 \geq C > 0 \\ 12 - \frac{1}{25}C & \text{if } 300 \geq C > 50 \end{cases}$$

From these equations, we can see that the market demand curve will have a vertical intercept of \$20, a kink at the point $(50, \$10)$, and a horizontal intercept of 300 cups of coffee. The slope of the upper segment of the demand curve will be $-\frac{1}{5}$ and the slope of the lower segment will be $-\frac{1}{25}$.

(d) Using the market demand function you found in part (c), is the price elasticity of demand for the market as a whole at a price of \$4 larger or smaller than the elasticities you found for the individual consumers? Explain why your result makes intuitive sense (in other words, explain why you could have predicted that result even without calculating the elasticity).

First note that at a price of \$4, we are on the lower segment of the demand curve and should use the corresponding equation of $C(p) = 300 - 25p$. Using this demand equation, the elasticity will be:

$$\varepsilon_{\text{market}} = \frac{p}{C(p)} \frac{dC(p)}{dp}$$

$$\varepsilon_{\text{market}} = \frac{p}{300 - 25p}(-25)$$

$$\varepsilon_{\text{market}} = \frac{4}{300 - 25 \cdot 4}(-25)$$

$$\varepsilon_{\text{market}} = -\frac{1}{2}$$

This elasticity is in between the elasticities for the individual consumers. This makes sense when we recognize that elsticity is giving us the percent change in demand with a one percent change in quantity. So with an elasticity of $-\frac{2}{3}$ for type A consumers, C_A will go up by $\frac{2}{3}$ of a percent when price falls by one percent. When we have 10 of these consumers, their overall demand will increase by $\frac{2}{3}$ of a percent (since we're dealing with percentages, adding more of the same consumer type will lead to a bigger overall change in quantity but the percent change will remain the same). The other half of the consumers have a lower elasticity of only $-\frac{1}{4}$, so their demand will only change by a quarter of a percent. The overall change in market demand can be though of as a weighted average of these two consumer groups, leading to a percent change that is an average of two groups' individual percent changes.

5. (15 points) Consider the market for a life-saving drug. Since the drug is life-saving, demand for the drug does not depend on price. Demand is always equal to 500 units no matter what the price is. The supply curve is an upward sloping line.

   (a) On a graph with quantity on the horizontal axis and price on the vertical axis, show the demand curve and the supply curve assuming that the equilibrium price for the drug is \$20.

   Your graph should show a vertical demand curve at a quantity of 500. The supply curve should be an upward sloping line passing through the point $(500, \$20)$ (intersecting the demand curve at that point).

   (b) On the same graph you drew above, show the change in consumer surplus, change in producer surplus and tax revenue resulting from a quantity tax of \$5 a unit placed on suppliers. Whenever possible, label these items with their numerical values. (Note: If one of these items does not exist, write that beside the graph. For example, if there was no tax revenue you would not be able to show that on the graph. Instead you would just write "tax revenue = 0" on the side.)

   If the tax is placed on suppliers, we can find the new equilibrium price and quantity by shifting the supply curve up by the amount of the tax. The price at which this shifted supply curve intersects the demand curve will be the new price paid by consumers. The price received by suppliers net of the tax is just this new equilibrium price minus the amount of the tax. Given that the demand curve is vertical, the quantity at which this shifted supply curve intersects the demand curve will still be at 500 units. The new price paid by consumers will therefore just be the original \$20 plus the tax, so \$25. The price received by the sellers will be \$25 minus the \$5, or \$20 (the same price they received before the tax).

   As for the changes in consumer surplus, producer surplus and tax revenue, producer surplus does not change because the firms sell the same quantity and the price they receive net of the tax is the same as the price they received before. Consumer surplus is reduced by the amount of the tax times the quantity of 500 (since the price has increased for consumers by \$5 for each of those 500 units). Tax revenue is equal in magnitude to this change in consumer surplus as it is just the quantity sold times the tax per unit.

   (c) Which of the values from part (b) would change if the same tax were placed on consumers rather than suppliers?

   Whether the tax is placed on consumers or producers, the outcome will be the same. Quantity will remain at 500 because demand is perfectly inelastic. Also because of the perfectly inelastic demand, the entire tax will be passed along to consumers, reducing consumer surplus by 500 times the \$5 tax, a reduction equal in magnitude to the tax revenue raised.