Econ 303 - Intermediate Microeconomic Theory
College of William and Mary February 21, 2014
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Midterm 1 - Solutions

You have until 1:50pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that individuals can consume fractions of units. Remember to put your name on the exam. Good luck!

Name:

1. (25 points) Anthony enjoys baking biscuits. His biscuit recipe requires one part buttermilk to two parts flour. So if Anthony uses two cups of buttermilk, he must use four cups of flour. If he uses half a cup of buttermilk, he must use one cup of flour. Anthony's utility increases with the number of biscuits he is able to bake.

   (a) Write down a utility function that gives Anthony's utility in terms of cups of buttermilk (B) and cups of flour (F).

   Since Anthony gets utility from biscuits and the number of biscuits will be limited by whichever ingredient is in short supply, his utility is can be represented with a min function of the following form:

   $$U(B, F) = \min(\alpha B, \beta F)$$

   The issue now is what values to use for the coefficients α and β. Note that Anthony needs twice as much flour as buttermilk, so he will be constrained by whichever is smaller, the amount of buttermilk he has or one half the amount of flour. We need our coefficients to reflect this: β should be half the size of α. One such choice of coefficients would be:

   $$U(B, F) = \min(2B, F)$$

   (b) Suppose that a cup of buttermilk costs \$4 and a cup of flour costs \$1. If Anthony has \$100 to spend, how many cups of buttermilk will Anthony buy and how many cups of flour will he buy?

   We cannot use our tangency condition here because we can't get a ratio of marginal utilities with the min function. So we need to take a step back and think about how Anthony will maximize his utility. Note that any extra buttermilk or flour simply goes to waste. Therefore Anthony will maximize the utility he gets from his \$100 by purchasing buttermilk and flour in the exact right combination. This gives us our first equation describing Anthony's optimal bundle:

   $$2B = F$$

telling us that Anthony should buy twice as much flour as buttermilk. The other condition that will pin down Anthony's optimal bundle is his budget constraint. He should spend all of his money:

$$p_B B + p_F F = I$$

Plugging in $2B$ for F in this equation and solving for B gives us:

$$p_B B + p_F \cdot 2B = I$$

$$B(p_B + 2p_F) = I$$

$$B = \frac{I}{p_B + 2p_F}$$

This is our demand equation for buttermilk. To get the demand equation for flour, we can simply plug this result back into our equation relating B to F:

$$F = 2B$$

$$F = \frac{2I}{p_B + 2p_F}$$

Now it is simply a matter of plugging in the appropriate prices and income to determine how much buttermilk and flour Anthony will buy:

$$B = \frac{100}{4 + 2 \cdot 1} = \frac{50}{3}$$

$$F = \frac{2 \cdot 100}{4 + 2 \cdot 1} = \frac{100}{3}$$

(c) If the price of flour rises to \$2 a cup, how much will Anthony's demand for flour change? You should be able to give an exact number.

Now that we have our demand functions, we can simply plug in the new price to get the new quantity:

$$F = \frac{2 \cdot 100}{4 + 2 \cdot 2} = 25$$

(d) Use a graph to decompose this change in flour into the change due to the income effect and the change due to the substitution effect. Be certain to clearly label all relevant budget lines, indifference curves and bundles, using numerical values when possible.

Before getting to the graph, let's finish getting the relevant bundles. We already know the original bundle from part (b). We got the amount of flour in the new bundle in part (c). The amount of buttermilk in this bundle will be:

$$B = \frac{100}{4 + 2 \cdot 2} = \frac{25}{2}$$

Now we need the intermediate bundle that corresponds to a budget line with the new prices but an income adjusted to keep the original bundle just affordable. This income will be what the original bundle costs using the new prices:

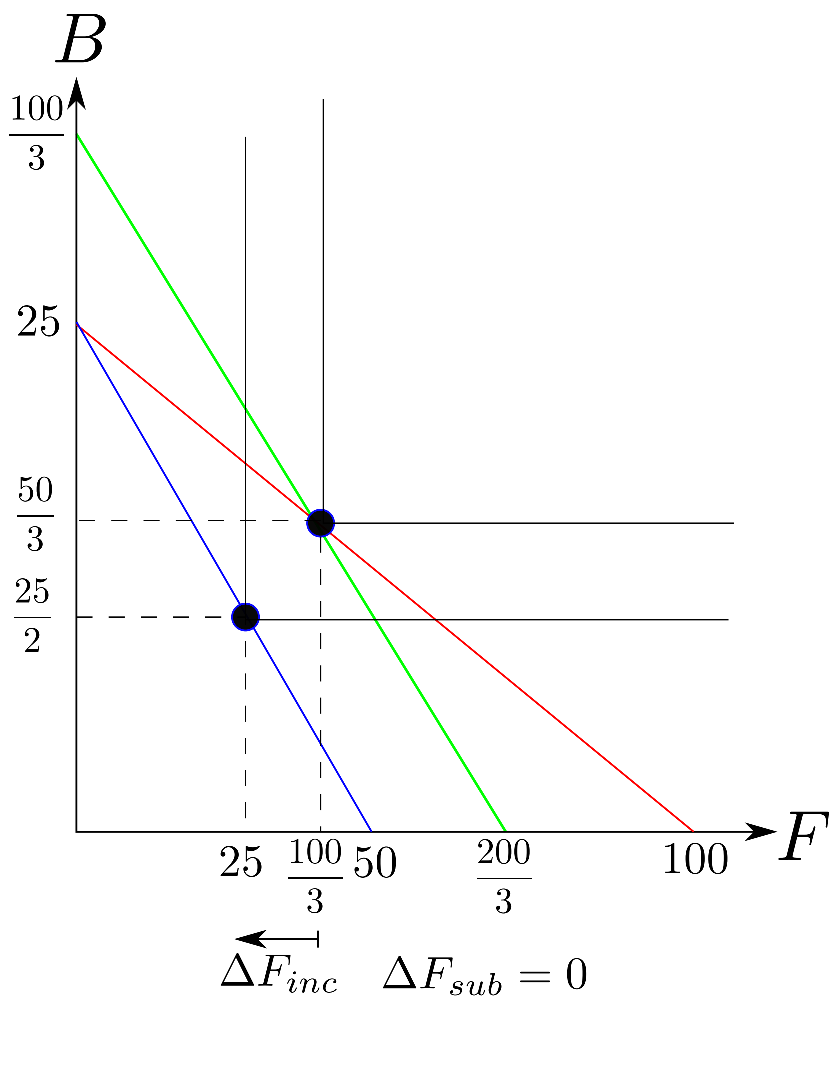
$$\tilde{I} = p_B \cdot B_a + p'_F \cdot F_a$$

$$\tilde{I} = 4 \cdot \frac{50}{3} + 2 \cdot \frac{100}{3}$$
$$\tilde{I} = \frac{400}{3}$$

Plugging this income and the new prices into our demand equations will give us the intermediate bundle:

$$B = \frac{\frac{400}{3}}{4 + 2 \cdot 2} = \frac{50}{3}$$
$$F = \frac{2 \cdot \frac{400}{3}}{4 + 2 \cdot 2} = \frac{100}{3}$$

The substitution effect is the movement from the original bundle to this intermediate bundle (the change in demand caused by the change in relative prices holding effective income constant). The income effect is the movement from the intermediate bundle to the final bundle (the change in demand caused by the change in effective income holding relative prices constant). These changes are shown on the graph below.



Notice that there actually is no substitution effect in this case, only an income effect. The original bundle and the intermediate bundle are actually the same bundle. This seems quite unusual but makes sense once you think about the indifference curves in this case. We are dealing with perfect complements which give us L-shaped indifference curves. When we change the relative prices, rotating the budget line, but keep the budget line passing through the original bundle, the original bundle remains the optimal bundle. Graphically, this is because we remain at the kink in the indifference curve; any budget line passing through the point is effectively 'tangent' to the L-shaped indifference curve. Conceptually, what's going on is that we need to combine our goods in a fixed proportion. Changing the relative prices doesn't change that desired proportion, so there ends up being no substitution effect.

2. (25 points) For each scenario below, write down an equation capturing Betsy's budget constraint and graph Betsy's budget line. For the budget equation, use numerical values wherever possible. For the graph, be certain to clearly label the axes and label all slopes, intercepts and kinks with their numerical values if possible. Use a separate graph for each scenario.

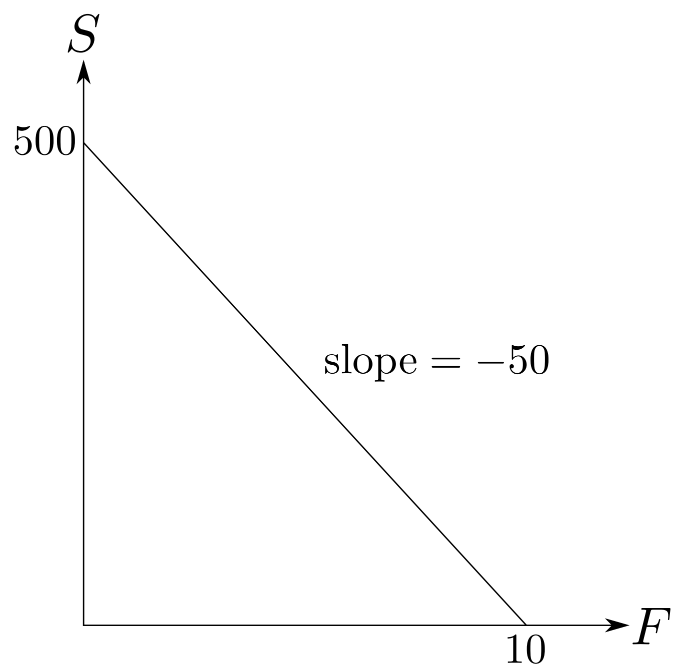
   (a) Betsy has 5,000 frequent flyer miles. These miles can be redeemed for flights (F), with 500 miles needed for each flight, or they can be used for song downloads (S), with 10 miles needed for each song download.

   In this case, Betsy's 'income' is 5,000 miles, the price of a flight is 500 miles and the price of a song is 10 miles, so the budget equation is a fairly standard budget equation of the following form:

$$p_F F + p_S S = I$$

$$500F + 10S = 5000$$

   The graph of this budget equation is a straight line that intersects the flight axis at 10 and the song axis at 500, as shown below.



   (b) Betsy has joined a gym. With her membership she gets four free yoga classes (Y) each month. She needs to pay for each yoga class after that and for each zumba class (Z) she takes at the gym. Yoga classes and zumba classes each cost \$5 a class. Betsy's monthly budget for classes is \$100.

   First let's think about the budget constraint if Betsy takes more than four yoga classes. This would be:

$$p_Y(Y - 4) + p_Z Z = I$$

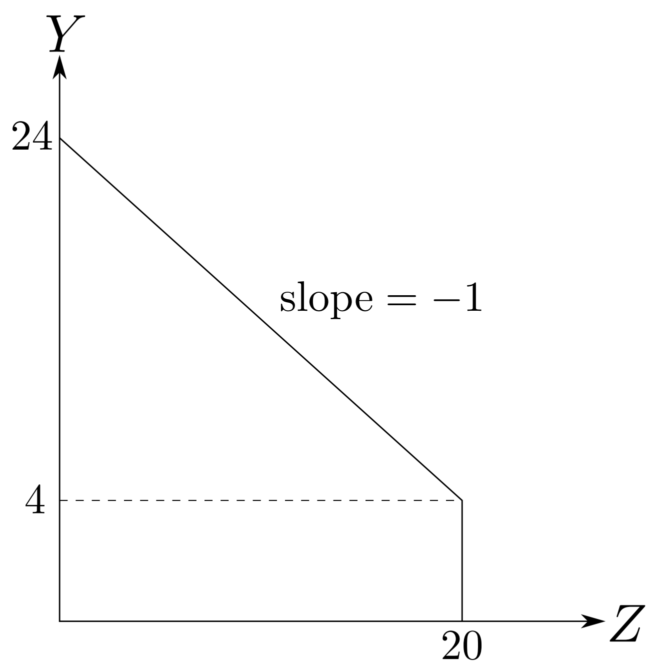
$$5(Y-4)+5Z=100$$

The reason is that we are using $Y-4$ is that Betsy is only paying for the yoga classes after the first four. This portion of the budget line is just a standard budget line with a slope of -1 going from 24 yoga classes and no zumba classes to 4 yoga classes and 20 zumba classes.
Now let's consider her budget line if she takes fewer than four yoga classes. In this case, all of her yoga classes are covered by the free classes, so she can spend all of her money on zumba classes, giving a budget line of:

$$p_Z Z = I$$

$$5Z = 100$$

This segment of the budget line is simply a vertical line at $Z = 20$.



(c) Betsy buys fruit (F) at Farm Fresh and vegetables (V) at Food Lion. Farm Fresh charges \$1 for each piece of fruit. Food Lion charges \$2 for each vegetable. Farm Fresh has a special promotion where you receive 20% off of your entire fruit order when you spend \$20 or more (before the discount is applied). Betsy has a total of \$100 to spend on fruit and vegetables.

There will be two different segments of Betsy's demand curve, one where she spends less than \$20 and one where she spends more than \$20 (pre-discount) on fruit. Note that when she is spending exactly \$20 on fruit, she is spending \$80 on vegetables which buys 40 vegetables. So these two demand segments correspond to when she is buying more than 40 vegetables and less than 40 vegetables, respectively. For the first of these segments, her budget is:

$$p_V V + p_F F = I \text{ if } V > 40$$

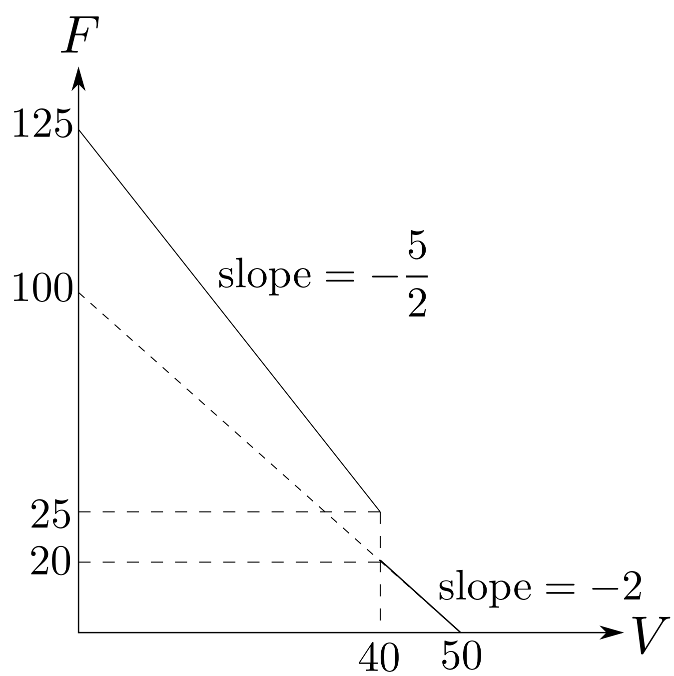
$$2V + F = 100 \text{ if } V > 40$$

Once she spends \$20 (pre-discount) on fruit she gets a 20% discount on fruit. So the price of fruit is reduced by 20% to $\frac{4}{5}$. Note that this applies to all of her fruit purchase, so the new budget equation is:

$$p_V V + p'_F F = I \text{ if } V \leq 40$$

$$2V + \frac{4}{5}F = 100 \text{ if } V \leq 40$$

Graphing these two different segments of the budget constraint gives us:



(d) Betsy has two hours between classes to respond to emails (E) and browse websites (W). Each website takes ten minutes to browse. If Betsy spends all of her time responding to emails, she can get through fifty emails in the two hours. As she responds to more emails, she gets quicker at typing responses. So the tenth email response takes less time than the fifth email response did. *Note: there are multiple correct ways to write the budget constraint and draw the graph for this scenario, you just need to make sure your equation and graph are consistent with the information given.*

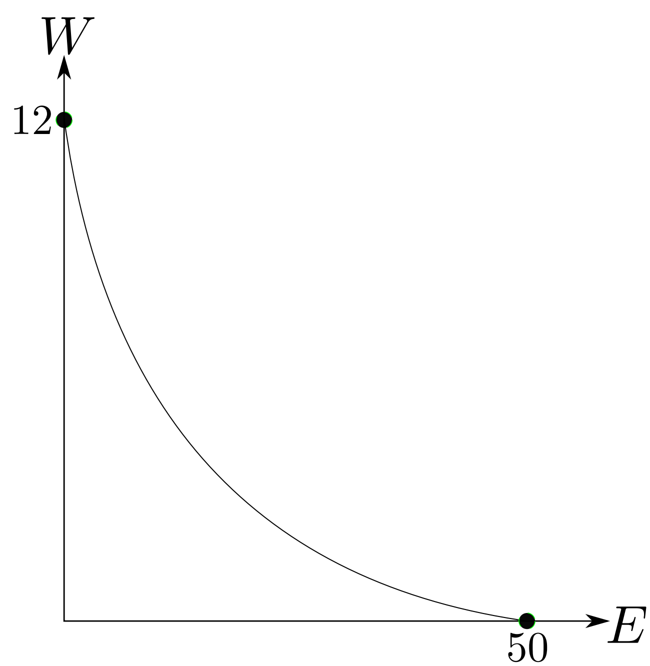
In this case we have an unusual budget constraint. We have a limited amount of time, 120 minutes, which is effectively our income. Web browsing takes ten minutes per page, so the time left for responding to emails will be 120 minus $10W$. That is all fairly standard. What is strange in this case is that the price of responding to an email, in terms of how much time is costs us, is changing. As we lower the number of web pages we visit the number of emails we can respond to is increasing but *at an increasing rate*. So our budget constraint is nonlinear. It starts out steep when we are slow at responding to emails and then gets flatter and flatter as we get better and better at responding to emails. What is a little more obvious is the endpoints of the budget constraint. If we spend all our time browsing websites, we'll get to 12 websites (10 minutes per site for 120 minutes). If we spend all of our time responding to email we will get through 50 emails. This gives us our two endpoints for the budget constraint. The constraint itself is a convex curve connecting these two points. The budget equation will also need to be nonlinear for the email term:

$$p_W W + p_E(E)E = 120$$

Notice that the price of an email is written as a function of E since it is decreasing as E increases. One way to do this is the following (note that I don't expect you to figure out an exact constraint in this case):

$$10W + \frac{10}{E^{0.36}}E = 120$$

Note that if I plug in zero for W and 50 for E, the left-hand-side gives us 120 minutes, matching the endpoint of the budget constraint.



3. (25 points) There are three types of consumers for hamburgers (H) and soda (S) with the following demand functions for hamburgers in terms of the price of a hamburger, p_H, and the price of a soda, p_S:

$$H_A(p_H, p_S, I_A) = \frac{I_A}{p_H + p_S} \tag{1}$$

$$H_B(p_H, p_S, I_A) = \frac{I_B}{p_H + 2p_S} \tag{2}$$

$$H_C(p_H, p_S, I_A) = \frac{I_C}{p_H + 3p_S} \tag{3}$$

where I_A, I_B and I_C are the individual incomes of consumers of type A, B and C, respectively. There are five consumers of each type for a total of fifteen consumers in the market.

(a) Based on the demand functions above, determine whether hamburgers are normal or inferior, whether they are ordinary or Giffen, and whether soda is a substitute or complement. Be certain to fully justify your answers.

Notice that when p_H increases in any of the three equations, it increases the denominator making the overall value of the demand function decrease. Demand for hamburgers decreasing when the price of hamburgers increases tells us that hamburgers are ordinary goods. When I increases in any of the demand functions, the function gets larger, implying that hamburgers are normal goods. Finally, note that in all three demand functions, increasing p_S increases the denominator, decreasing the value of the function. Therefore an increase in the price of soda decreases demand for hamburgers, implying the two are complements.

(b) Will the market demand curve have kinks in it? If so, at what prices will these kinks occur? If not, explain why there are no kinks.

There will be no kinks for the market demand curve because all consumers buy positive quantities of hamburgers at all values of p_H (no matter how large p_H gets, $\frac{I_n}{p_H + n \cdot p_S}$ is always a positive number). There is no price at which an additional customer enters the market, so we do not get any kinks.

(c) Suppose that the price of a soda is \$2 and that the income of a type A consumer is \$100, the income of a type B consumer is \$150 and the income of a type C consumer is \$200. Derive the an expression for the aggregate market demand for hamburgers as a function of the price of hamburgers.

We already noted above that all consumers buy positive quantities of hamburgers at all values of p_H so we do not need to worry about multiple segments of the market demand curve. The market demand curve is therefore just the sum of all of the individual demands. This is:

$$H_{market} = 5H_A + 5H_B + 5H_C$$

We are multiplying each individual demand by five because there are five of each customer type. Plugging in the appropriate functions and values for parameters gives us:

$$H_{market} = 5 \cdot \frac{I_A}{p_H + p_S} + 5 \cdot \frac{I_B}{p_H + 2p_S} + 5 \cdot \frac{I_C}{p_H + 3p_S}$$
$$H_{market} = 5 \cdot \frac{100}{p_H + 2} + 5 \cdot \frac{150}{p_H + 2 \cdot 2} + 5 \cdot \frac{200}{p_H + 3 \cdot 2}$$
$$H_{market} = \frac{500}{p_H + 2} + \frac{750}{p_H + 4} + \frac{1000}{p_H + 6}$$

This is as far as we can simplify the market demand function.

(d) Let's simplify things and assume that there is only one consumer of hamburgers and that consumer is of type A. The consumer's income is \$100 and the price of a soda is still \$2. Find an expression for the elasticity of demand for hamburgers with respect to price and use it to determine whether a restaurant's revenues from burgers will increase or decrease if they raise the price of a hamburger by a small amount. You expression for the elasticity should contain only numerical constants and the price of hamburgers.

Elasticity will be given by:

$$\epsilon = \frac{p_H}{H_A(p_H, p_S, I_A)} \frac{dH_A}{dp_H}$$

$$\epsilon = \frac{p_H}{\frac{I_A}{p_H + p_S}} \frac{-I_A}{(p_H + p_S)^2}$$
$$\epsilon = -\frac{p_H}{p_H + p_S}$$

Plugging in the price of soda gives us:

$$\epsilon = -\frac{p_H}{p_H + 2}$$

Notice that this is smaller in magnitude than one so demand for hamburgers is inelastic. This tells us that revenue will increase with a small price increase (the revenue lost from fewer sales will be smaller than the revenue gained from charging more for the remaining sales because demand is not very responsive to price).

4. (25 points) Clyde's marginal utility from watching hockey games, MU_H, and his marginal utility from watching curling matches, MU_C, are given by:

$$MU_H = 20HC^{\frac{1}{2}} \tag{4}$$

$$MU_C = \frac{5H^2}{C^{\frac{1}{2}}} \tag{5}$$

where H is the number of hockey games he watches and C is the number of curling matches he watches. Each hockey game takes two hours to watch. Each curling match takes one hour to watch. Clyde's has T hours of total time to watch hockey and curling.

(a) Determine which of the following properties hold for Clyde's preferences, giving a brief explanation in each case: diminishing marginal rate of substitution, monotonicity, convexity.

Let's begin with monotonicity. Both marginal utilities will be positive for any positive quantities. Therefore more of each good always raises utility and we have monotonic preferences. Next let's consider the marginal rate of substitution (for a graph with H on the horizontal axis and C on the vertical axis):

$$MRS = -\frac{MU_H}{MU_C}$$

$$MRS = -\frac{20HC^{\frac{1}{2}}}{\frac{5H^2}{C^{\frac{1}{2}}}}$$

$$MRS = -\frac{4C}{H}$$

Notice that as we move down and right along an indifference curve, increasing H and decreasing C, this gets smaller in magnitude so we have a diminishing marginal rate of substitution. Given that the indifference curves get flatter as we move to the right, they will be convex: a bundle that is an average of two bundles lying on the same indifference curve will lie above that indifference curve.

(b) Derive an expression for the amount of hockey Clyde will watch as a function of his total time T and an expression for how much curling he will watch as a function of T.

Clyde will want to split his time such that the marginal utility from an extra hour spent watching hockey is just equal to the marginal utility from another hour spent watching curling. If this weren't true, he could reallocate time to the higher marginal utility per hour sport to increase his overall utility. This optimal combination of hockey and curling will be where the ratio of the marginal utilities is just equal to the ratio of prices in terms of how much time each event takes, our standard tangency condition:

$$\frac{MU_H}{MU_C} = \frac{p_H}{p_C}$$

$$\frac{20HC^{\frac{1}{2}}}{\frac{5H^2}{C^{\frac{1}{2}}}} = \frac{p_H}{p_C}$$

$$\frac{4C}{H} = \frac{p_H}{p_C}$$

$$C = \frac{p_H}{4p_C}H$$

Now we can plug this into Clyde's budget constraint:

$$p_C C + p_H H = T$$

$$p_C\left(\frac{p_H}{4p_C}H\right) + p_H H = T$$

$$\frac{5}{4}p_H H = T$$

$$H = \frac{4T}{5p_H}$$

Plugging this back into the result from our tangency condition will give us the demand equation for curling:

$$C = \frac{p_H}{4p_C}\frac{4T}{5p_H}$$

$$C = \frac{T}{5p_C}$$

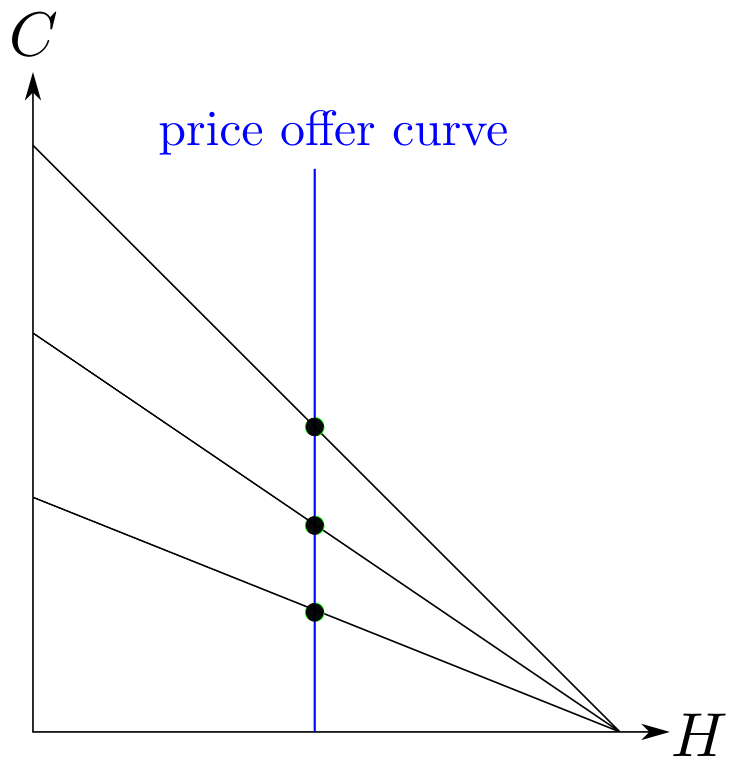
Plugging in the values for p_H (two hours) and p_C (one hour) gives us:

$$H = \frac{2}{5}T$$

$$C = \frac{1}{5}T$$

(c) On a graph with hockey on the horizontal axis and curling on the vertical axis, show the price offer curve generated by varying the amount of time a curling match takes (you can think of this as NBC deciding to edit curling matches so they take less than one hour, this editing does not affect Clyde's enjoyment of the match). Show at least three points on the price offer curve and the corresponding budget lines and indifference curves. There is no need for numerical values but the shapes of the curves should be consistent with the information given in the problem.

The price offer curve traces out the optimal bundles as the price of curling decreases. Notice that the price of curling did not enter into our demand for hockey. So the value of H in the optimal bundle will be the same all along the price offer curve. Looking at our demand equation for curling, as p_C decreases, C will increase. So as we lower p_C, rotating the budget line up, the amount of H will stay the same while the amount of C will increase, generating a vertical price offer curve.


C
price offer curve
H