# Midterm 1 - Solutions

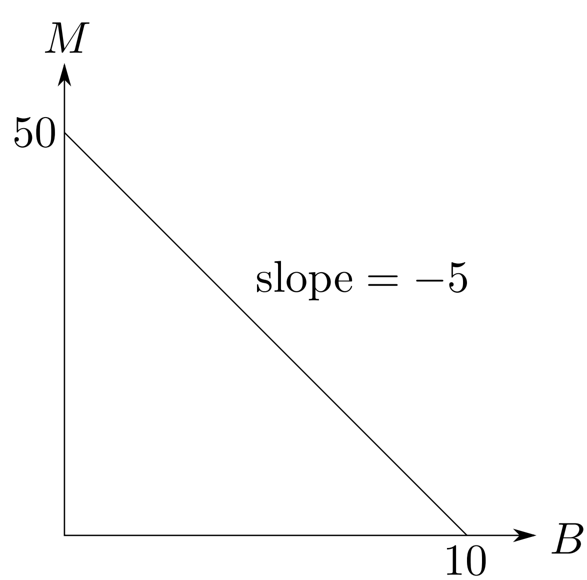
You have until 1:50pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can produce fractions of units and charge non-integer prices (so a firm could produce 82.4 units and sell at a price of $5.325 per unit). Remember to put your name on the exam. Good luck!

**Name:** **ID Number:**

1. (25 points) Arnold consumes only books and magazines. For each scenario below, draw a graph of Arnold's budget constraint. You should use a separate graph for each scenario. In each case, books should be on the horizontal axis and magazines should be on the vertical axis. You must label all endpoints, kinks and slopes with their numerical values for full credit:

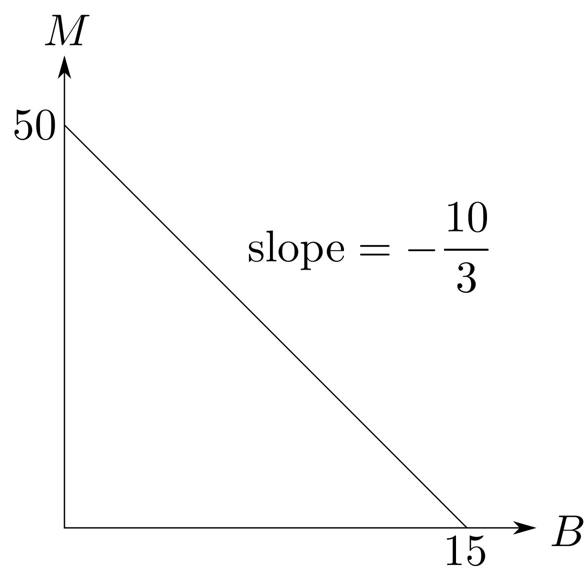
   (a) Arnold has $100 to spend. The price of a book is $10 and the price of a magazine is $2.

   This is simply a standard budget line. If Arnold spends all of his money on magazines, he can afford 50 of them ($100 in income divided by $2 per magazine). Therefore the vertical intercept of the budget line should be at 50. If he spends all of his money on books he can afford 10 of them ($100 divided by $10 per book) giving us a horizontal intercept of 10. The slope of the budget line will be $-\frac{p_B}{p_M}$ which is $-5$.



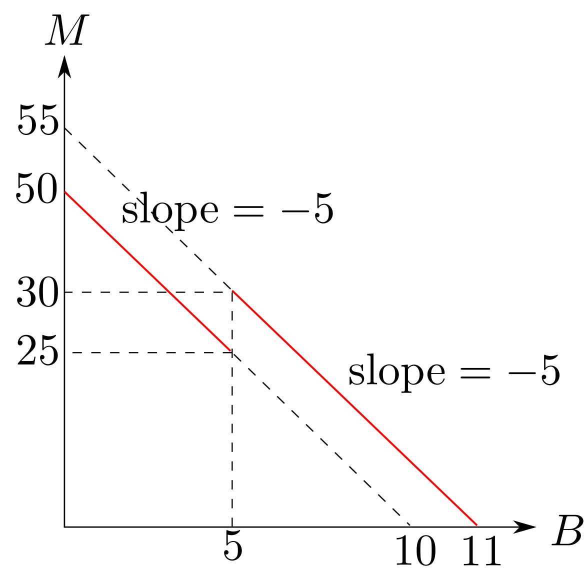
(b) The prices and income are the same as in part (a) only now the bookstore runs a special where books are three for the price of two. (Note: This special works for any number of books. For example, you would get 1.5 books for the price of one.)

This new deal effectively changes the price of books. Rather than two books for \$20, Arnold now gets three books for \$20. So the new effective price of a single book is \$20 divided by three books or $\$\frac{20}{3}$ per book. This does not change the vertical intercept of the budget line as magazines still cost the same amount. However, it does change the horizontal intercept and the slope. The new horizontal intercept is \$100 divided by $\$\frac{20}{3}$, or 15 books. The new slope is $-\frac{\frac{20}{3}}{2}$, or $-\frac{10}{3}$.




(c) The prices and income are the same as in part (a) only now the bookstore will take 10% off of Arnold's entire purchase, both books and magazines, if he spends at least \$50 on books.

This last one is a bit trickier. If Arnold is spending less than \$50 on books, in other words if he is buying fewer than 5 books, his budget line looks exactly the same as in part (a). So the first segment of Arnold's budget line will be a straight line with a slope of −5 going from 50 magazines and no books to 25 magazines and 5 books. Once Arnold buys at least five books, the bookstore will lower the prices of both books and magazines by taking 10% off the entire order. Notice that this does not change the relative prices of books and magazines: if both prices are reduced by 10%, a book will still cost five times as much as a magazine. So the slope of the second segment of the budget line will still be −5. What the 10% is really doing is giving Arnold more income. He can now buy \$111.11 worth of books and magazines with his \$100 (when they take 10% off of the \$111.11, his final bill will be \$100). So the second segment of the budget line is the budget line corresponding to an income of \$111.11 with the price of magazines being \$2 and the price of books being \$10. Note that for the graph, I have rounded this income to \$110 to make the numbers simpler to read, you will receive full credit whether you use the exact \$111.11 (giving endpoints of 55.56 magazines and 11.11 books) or the rounded \$110 (giving endpoints of 55 magazines and 11 books). Arnold's budget line is shown in red on the graph below (the dashed lines are not part of the budget line).



2. (25 points) Bob's preferences for chocolate and bananas exhibit the following properties:

   - The marginal utility of chocolate and the marginal utility of bananas are both always positive.
   - Bob's optimal bundle always contains positive quantities of both chocolate and bananas.
   - Chocolate and bananas are complements.
   - Bob's income offer curve has a positive slope.

   On a graph with chocolate on the horizontal axis and bananas on the vertical axis, show the change in chocolate due to the substitution effect and the change in chocolate due to the income effect when the price of bananas decreases. Label the change in chocolate due to the income effect $\Delta C_{inc}$ and the change due to the substitution effect $\Delta C_{sub}$ Be certain to clearly label all relevant consumption bundles, budget lines and indifference curves. If you need to make any additional assumptions about the two goods, state them clearly and succinctly. (Note that you will not be able to provide numerical values for anything on the graph.)

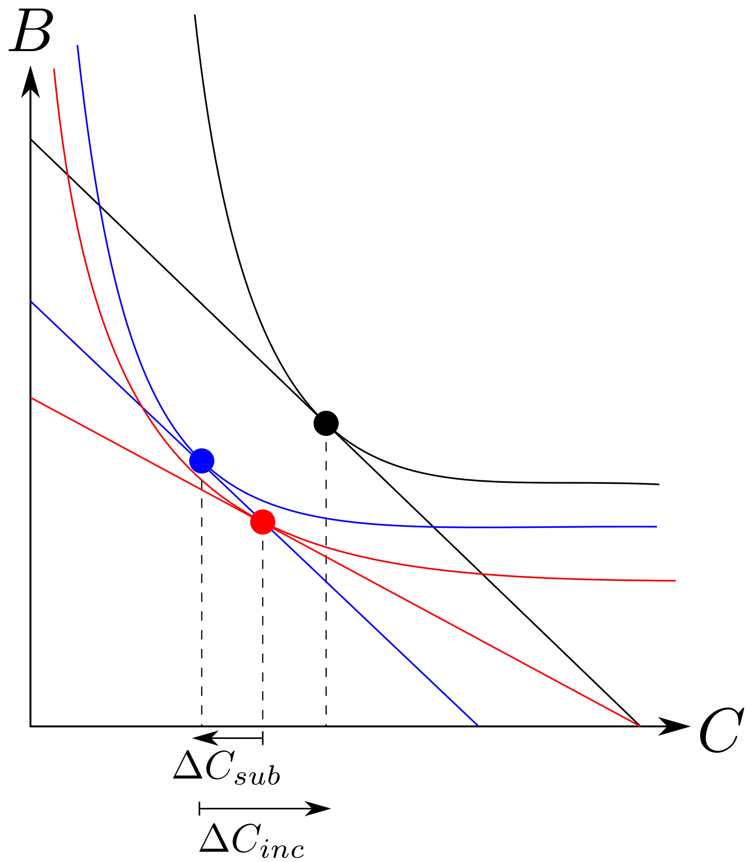
   The description of Bob's preferences tells us several things about how the graph should look:

   - Because the marginal utilities are always positive, Bob will always spend all of his money. This means that all of the optimal bundles will lie on, not below, their respective budget lines.
   - The optimal bundles always contain positive amounts of both goods so they will never be at the endpoints of the budget lines.
   - The fact that chocolate and bananas are complements tells us that the quantity of chocolate in the optimal bundle should increase when the price of bananas decreases.
   - The optimal bundle after the price change should have more chocolate and more bananas than the intermediate bundle because both goods are normal, suggesting that an increase in income will increase demand for both.

   Given that the price of bananas went down, the intermediate bundle should have more bananas and less chocolate than the original bundle (bananas become relatively cheaper, chocolate becomes relatively more expensive). From above, we know that the final bundle should have more chocolate and more bananas than both the original bundle (because the goods are complements and because bananas are ordinary) and the intermediate bundle (because both goods are normal and effective income has gone up). The graph below shows income and substitution effects consistent with these properties. The original budget line, optimal bundle and corresponding indifference curve are shown in red. The intermediate budget line, optimal bundle and corresponding indifference curve are shown in blue. The final budget line, optimal bundle and corresponding indifference curve are shown in black. The key features of this graph are the following:

   - The intermediate bundle is to the left and above the original bundle.
   - The intermediate bundle is to the left and below the final bundle.
   - The final bundle is above and to the right of the original bundle.

- The substitution effect for chocolate is negative.
- The income effect for chocolate is positive and larger than the substitution effect.



3. (30 points) Christine's marginal utility from oranges, $MU_O$, and her marginal utility from kiwis, $MU_K$, are given by the following functions:

$$MU_O(O,K) = \frac{4}{O^{\frac{1}{2}}} \tag{1}$$

$$MU_K(O,K) = \frac{1}{K^{\frac{1}{2}}} \tag{2}$$

(a) Derive an expression for Christine's demand for kiwis as a function of her income and the prices of oranges and kiwis: $K(I, p_O, p_K)$.

Christine will adjust her spending on kiwis and oranges until the marginal utility of a kiwi relative to an orange is exactly equal to the price of a kiwi relative to an orange. If these ratios were not equal, Christine would be able to increase her utility by buying more of the good that has a greater marginal utility relative to its price. So one condition for Christine's optimal bundle is the following:

$$\frac{MU_O}{MU_K} = \frac{p_O}{p_K}$$

$$\frac{\frac{4}{O^{\frac{1}{2}}}}{\frac{1}{K^{\frac{1}{2}}}} = \frac{p_O}{p_K}$$

Rearranging this equation will give us an expression for how many kiwis Christine will buy relative to oranges:

$$4\frac{K^{\frac{1}{2}}}{O^{\frac{1}{2}}} = \frac{p_O}{p_K}$$

$$K^{\frac{1}{2}} = \frac{1}{4}\frac{p_O}{p_K}O^{\frac{1}{2}}$$

$$K = \frac{1}{16}\frac{p_O^2}{p_K^2}O$$

To get $K$ and $O$ in terms of income and prices, we can plug this expression into the budget constraint:

$$p_O O + p_K K = I$$

$$p_O O + p_K \cdot \frac{1}{16}\frac{p_O^2}{p_K^2}O = I$$

$$O = \frac{I}{p_O + \frac{1}{16}\frac{p_O^2}{p_K}}$$

This is the demand function for oranges. To get the demand function for kiwis we can plug this result back into our equation relating $K$ to $O$:

$$K = \frac{1}{16}\frac{p_O^2}{p_K^2}\left(\frac{I}{p_O + \frac{1}{16}\frac{p_O^2}{p_K}}\right)$$

$$K = \frac{I}{16\frac{p_K^2}{p_O^2} \cdot (p_O + \frac{1}{16}\frac{p_O^2}{p_K})}$$

$$K = \frac{I}{16\frac{p_K^2}{p_O} + p_K}$$

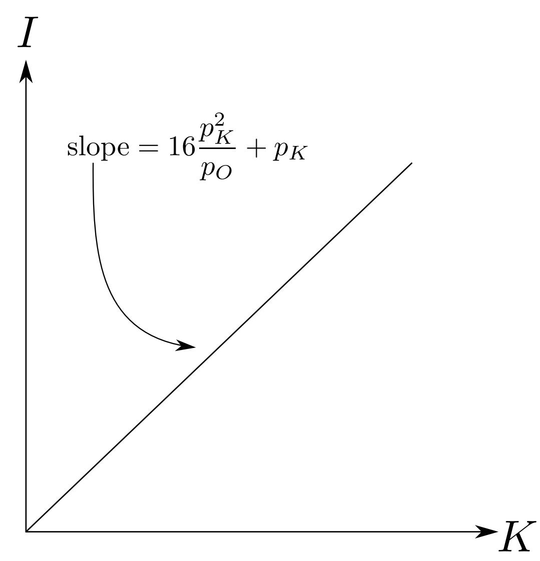
(b) Sketch the Engel curve and the demand curve for kiwis. Label any intercepts and kinks with their values in terms of $I$, $p_O$ and $p_K$. Also label the slopes of any linear segments on these graphs with their values in terms of $I$, $p_O$ and $p_K$.

We can determine the shapes of the Engel curve and demand curve for kiwis from the demand function for kiwis that we found in part (a). Let's begin with the Engel curve. This graph represents the relationship between income and demand for kiwis, with income on the vertical axis and kiwis on the vertical axis. We can rearrange the demand function to get the equation for the Engel curve:

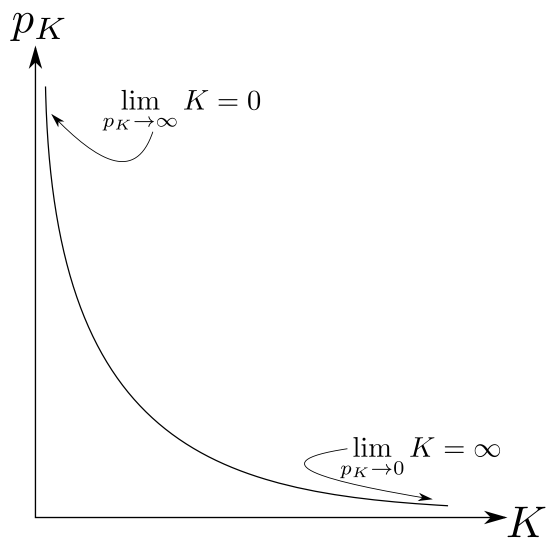
$$K = \frac{I}{16\frac{p_K^2}{p_O} + p_K}$$

$$I = \left(16\frac{p_K^2}{p_O} + p_K\right) K$$

Notice that $I$ and $K$ are linearly related and that when $K$ equals zero, $I$ equals zero. So the Engel curve will be a straight line passing through the origin. The slope is the coefficient in front of $K$ in the equation above: $16\frac{p_K^2}{p_O} + p_K$.




For the demand curve, the equation is far more complicated. It will be simpler to think about what happens in the limit where $p_K$ approaches infinity and the limit where $p_K$ approaches zero. As $p_K$ gets larger and larger, both terms in the denominator of the demand function for kiwis get larger while the numerator stays the same. This means that demand will approach zero. So as $p_K$ increases, the demand curve should approach the vertical axis. As $p_K$ gets closer to zero, both terms in the denominator of the demand function get closer to zero leading demand to get larger and larger. So as $p_K$ decreases, the demand curve should approach the horizontal axis.




(c) Christine gets 40 utils of utility from consuming the bundle (16 oranges, 16 kiwis). She also gets 40 utils of utility from the bundle (4 oranges, 144 kiwis). Would the bundle (10 oranges, 80 kiwis) give her more than 40 utils or less than 40 utils of utility? Be certain to fully justify your answer.

Notice that the bundle (10 oranges, 80 kiwis) is an average of the other two bundles. This bundle will provide more utility if the indifference curves are convex (an average of two bundles on an indifference curve will lie above that indifference curve) and will provide less utility if the indifference curves are concave (the average bundle would lie below the indifference curve). So to answer this question, we need to determine whether or not the indifference curves are convex. We can do this be looking at the marginal rate of substitution. On a graph with oranges on the horizontal axis and kiwis on the vertical axis, the marginal rate of substitution (and the slope of the indifference curves) will be:

$$MRS = -\frac{MU_O}{MU_K}$$

$$MRS = -\frac{\frac{4}{O^{\frac{1}{2}}}}{\frac{1}{K^{\frac{1}{2}}}}$$

$$MRS = -4\frac{K^{\frac{1}{2}}}{O^{\frac{1}{2}}}$$

Notice that as we move along an indifference curve from left to right, increasing $O$ and decreasing $K$, the magnitude of the $MRS$ is decreasing (the numerator is getting smaller while the denominator is getting larger). So the indifference curve is getting flatter when moving from left to right. This is a convex indifference curve. A bundle that is an average of any two bundles on that indifference curve will lie above the indifference curve and correspond to a higher level of utility. Therefore $(10 \text{ oranges}, 80 \text{ kiwis})$ will provide more than 40 utils of utility.

*Note: The approach above is certainly the most straightforward way to arrive at the correct answer. However, if you are someone who is more comfortable thinking about things in calculus terms there is an alternative way to approach this with some integration. We are given marginal utility functions but not a utility function. We could recover the utility function by integrating the marginal utility functions. First, note that the marginal utility for one of the goods does not depend on the level of the other good. So the two goods will appear in separate terms in the utility function. This let's us treat each term separately. The term of the utility function corresponding to $O$ will be:*

$$U_O = \int MU_O dO$$

$$U_O = \int \frac{4}{O^{\frac{1}{2}}} dO$$

$$U_O = 8O^{\frac{1}{2}}$$

*Following a similar approach for kiwis would give us the kiwi component of the utility function: $U_K = 2K^{\frac{1}{2}}$. So our utility function is:*

$$U(O, K) = U_O + U_K$$

$$U(O, K) = 8O^{\frac{1}{2}} + 2K^{\frac{1}{2}}$$

*Now we can simply plug the bundles into the utility function. First, note that the two original bundles do indeed give us a utility of 40 utils. Plugging in the third bundle gives us a utility of 43 utils.*

4. (20 points) Suppose that water and coffee are the only two beverages Donald can purchase. Donald likes water but each additional glass of water increases his utility by a smaller amount than the previous one did. Donald does not like coffee. Each cup of coffee he drinks reduces his utility by five units.

   (a) Write down a utility function that is consistent with Donald's preferences.

   There are multiple correct utility functions you could have provided. The key features your utility function must have are a positive, diminishing marginal utility for water and a negative, constant marginal utility for coffee equal to $-5$. One utility function meeting these criteria would be:

$$U(W, C) = W^{\frac{1}{2}} - 5C$$

   Notice that the marginal utilities based on this function are:

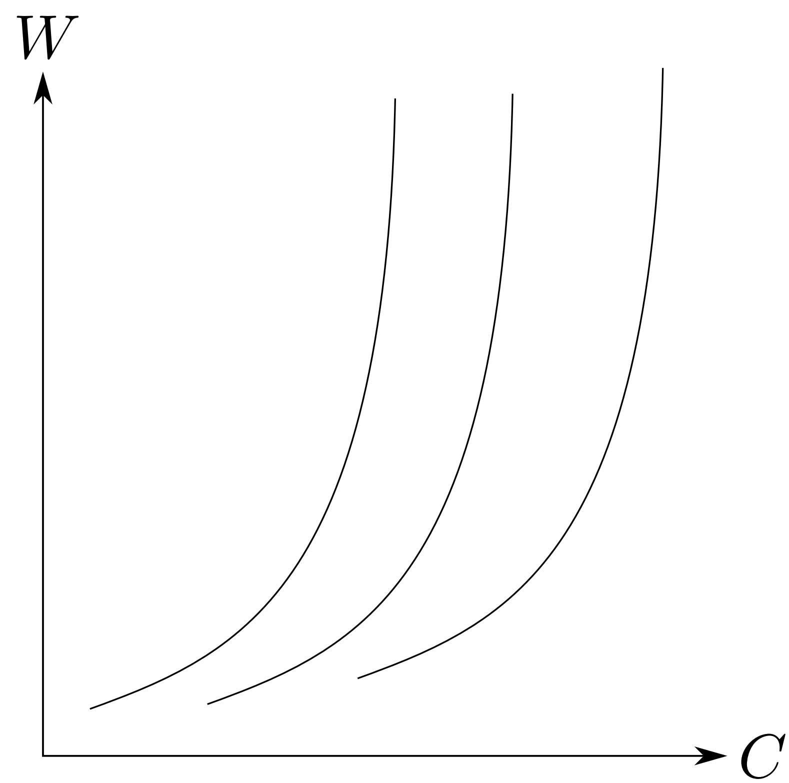
$$MU_W = \frac{1}{2}W^{-\frac{1}{2}}$$

$$MU_C = -5$$

   The marginal utility for water is always positive but gets smaller as $W$ gets smaller (any positive exponent less than one would have accomplished this). The marginal utility for coffee is negative, constant and equal to -5 just as we wanted.

   (b) On a graph with coffee on the horizontal axis and water on the vertical axis, sketch three indifference curves representing Donald's preferences.

   Because we have a good and a bad, the indifference curves should be upward sloping: if we give Donald more coffee it decreases his utility so we have to give him more water to bring him back up to his original utility level. Given that the marginal utility from water is diminishing the marginal utility from coffee is constant, it will require more and more glasses of water to compensate him for each additional cup of coffee as $W$ and $C$ go up. This implies that the indifference curves will be getting steeper as you move from left to right. A set of indifference curves consistent with these properties is shown below.




(c) Derive functions for Donald's demand for water as a function of income and prices, $W(I, p_W, p_C)$, and Donald's demand for coffee as a function of income and prices, $C(I, p_W, p_C)$.

This is actually more straightforward than the typical situation of deriving demand functions. Notice that additional coffee always reduces utility. Therefore it never makes sense to spend money on coffee rather than water. Donald should spend his entire income on water as additional water will always increase his utility. This gives us the following demand functions:

$$W(I, p_W, p_C) = \frac{I}{p_W}$$

$$C(I, p_W, p_C) = 0$$