Econ 303 - Intermediate Microeconomic Theory
College of William and Mary
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Midterm 1 - Solutions

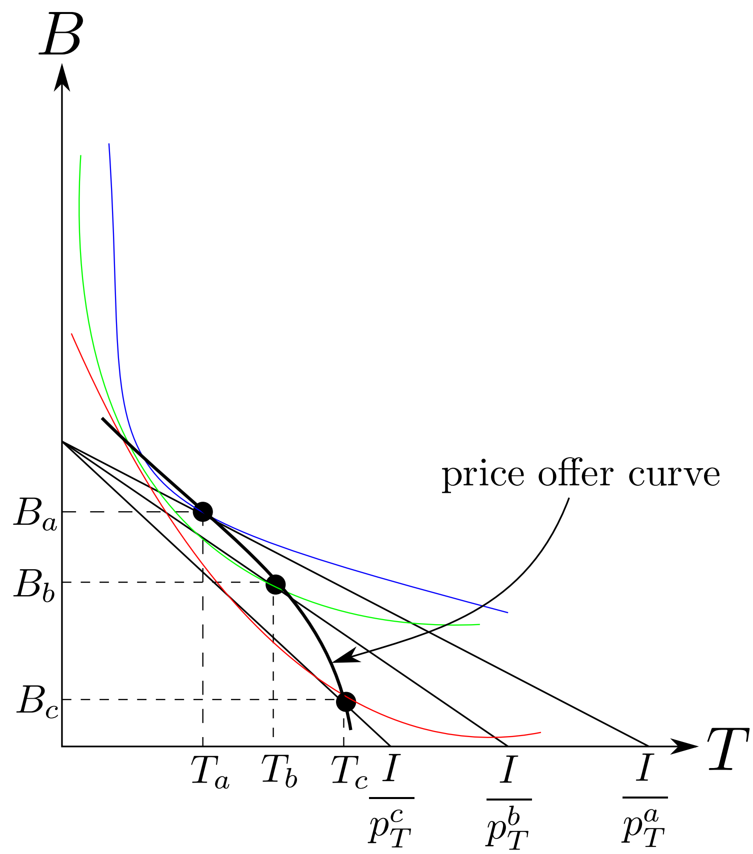
You have until 12:20pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Non-graphing calculators may be used (no graphing calculators or phones can be used). You may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can produce fractions of units and charge non-integer prices (so a firm could produce 82.4 units and sell at a price of $5.325 per unit). Remember to put your name on the exam. Good luck!

Name: **ID Number:**

1. (15 points) Suppose that tap water (T) and bottled water (B) are the only two goods a consumer buys and tap water is a Giffen good. Both the marginal utility of tap water and the marginal utility of bottled water are always positive.

   (a) Graph the price offer curve obtained by varying the price of tap water. Include all curves and lines necessary to show at least three points on the price offer curve and be certain to label everything clearly.

   The graph below shows the points on the price offer curve generated by increasing the price of tap water from p_T^a to p_T^b p_T^c. Because tap water is a Giffen good, the amount of tap water in the optimal bundle increases as the price of tap water decreases ($T_a < T_b < T_c$). Given that both goods provide positive marginal utility, we will always spend all of our income at the optimal bundle. If the quantity of tap water decreases as it gets cheaper, it implies we are spending less on tap water and must therefore be spending more on bottled water as the price of tap water decreases. This leads to increases in the amount of bottled water in the optimal bundle as p_T declines ($B_a > B_b > B_c$).



On your graph, you should have shown the budget line pivoting around the intercept on the bottled water axis as the price of tap water varied. Each optimal bundle should be shown as the point of tangency between the budget line and an indifference curve (note that the indifference curves should not intersect). The amount of tap water in the bundle should increase as the the price of tap water goes up and the amount of bottled water should decrease as the price of tap water increases.

(b) Use your graph and a written explanation to argue whether tap water and bottled water are complements or substitutes.

With both goods having positive marginal utility, the optimal bundle should always exhaust a consumers income (buying more of either good would increase utility if the individual was spending less than their total income). Spending on tap water will increase when the price of tap water increases because the quantity of tap water will increase (since tap water is a Giffen good). This means that spending on bottled water must go down when the price of tap water increases. So the quantity of bottled water in the optimal bundle decreases as the price of tap water increases. Therefore, by definition, bottled water is a complement.

2. (15 points) Each scenario below describes a different set of preferences for bundles of apples (A) and oranges (O). For each scenario, draw the indifference curve passing through the bundle (5 *apples*, 5 *oranges*) consistent with the description. Also give the numerical value of the slope of the indifference curve at the bundle (5 *apples*, 5 *oranges*).

   (a) Arnold always consumes apples and oranges together in fruit salad. His recipe for fruit salad calls for one orange for every two apples. He always follows the recipe and gets no enjoyment out of apples or oranges unless they are in fruit salad.

   Apples and oranges are always consumed in a fixed proportion making them perfect complements. Note that the number of fruit salads we can make is constrained by either the number of oranges Arnold has or one half the number of apples he has, whichever is smaller. So his utility function will be:

$$U(A,O) = \min(O, \frac{1}{2}A)$$

   Based on this utility function, the indifference curves will be L-shaped with the vertex of each indifference curve lying along the line $A = 2O$. Given that $\frac{1}{2}A < O$ at the bundle $(5,5)$, apples are the good limiting utility meaning the bundle is on the horizontal portion of the indifference curve, making the slope at that point zero.

   (b) Betsy like apples and dislikes oranges. Every apple she consumes increases her utility by ten utils while every orange she consumes decreases her utility by twenty utils.

   Marginal utility is constant for both goods so the marginal rate of substitution and therefore the slope of the indifference curves will be constant. The marginal rate of substitution is $-\frac{MU_O}{MU_A}$ which is $-\frac{(-20)}{10}$ or 2. So the indifference curve passing through the bundle $(5,5)$ should be an upward sloping line with a slope of 2.

   (c) Carl's utility from apples and oranges is given by the utility function $U(A,O) = A + O^2$.

   We can determine the shape and slope of the indifference curve using the marginal rate of substitution:

$$MU_A = \frac{dU}{dA} = 1$$

$$MU_O = \frac{dU}{dO} = 2O$$

$$MRS = -\frac{MU_O}{MU_A}$$

$$MRS = -2O$$

   This tells us that the indifference curve has a negative slope and that it is concave (as you move down and right to bundles with more O and less A, the magnitude of the MRS gets larger meaning a steeper indifference curve). To get the slope at the bundle $(5, 5)$, we simply need to plug in the values into the expression for the MRS:

$$MRS = -2 \cdot 5 = -10$$

So the slope will be -10 at that point.

3. (25 points) David's utility from shirts (S) and hats (H) is given by the following utility function:

$$U(S,H) = 5S^{\frac{3}{4}}H^{\frac{3}{4}} \tag{1}$$

His marginal utility from shirts and marginal utility from hats are given by:

$$MU_S(S,H) = \frac{15}{4}S^{-\frac{1}{4}}H^{\frac{3}{4}} \tag{2}$$

$$MU_H(S,H) = \frac{15}{4}S^{\frac{3}{4}}H^{-\frac{1}{4}} \tag{3}$$

(a) Assuming that David has $400 to spend on hats and shirts, derive an expression for David's optimal number of hats as a function of the price of hats and the price of shirts ($H(p_H, p_S)$).

We can find an expression for David's optimal quantity of hats by recognizing the at the optimal bundle, David will be spending his entire income and his indifference curve will be tangent to his budget line. Beginning with this tangency condition, we have:

$$\frac{p_H}{p_S} = \frac{MU_H}{MU_S}$$

$$\frac{p_H}{p_S} = \frac{\frac{15}{4}S^{-\frac{1}{4}}H^{\frac{3}{4}}}{\frac{15}{4}S^{\frac{3}{4}}H^{-\frac{1}{4}}}$$

$$\frac{p_H}{p_S} = \frac{H^{\frac{3}{4}}H^{\frac{1}{4}}}{S^{\frac{1}{4}}S^{\frac{3}{4}}}$$

$$\frac{p_H}{p_S} = \frac{H}{S}$$

$$S = \frac{p_H}{p_S}H$$

Plugging this into our budget constraint gives us:

$$p_S S + p_H H = I$$

$$p_S \frac{p_H}{p_S} H + p_H H = I$$

$$2p_H H = I$$

$$H = \frac{I}{2p_H}$$

(b) Based on your answer to part (a), determine whether hats are an ordinary good. Be certain to justify your answer.

Notice from the demand equation for H above, if the price of hats increases, demand for hats decreases. Technically, $\frac{dH}{dp_H} = -\frac{I}{2p_H^2} < 0$. So hats are an ordinary good.

(c) Suppose that the price of a shirt is \$10 and the price of a hat is \$10. What is the optimal number of hats?

$$H = \frac{I}{2p_H} = \frac{400}{2 \cdot 10} = 20$$

(d) The price of a shirt decreases to \$5 leading to a change in the combination of shirts and hats purchased. Decompose the change in the number of hats purchased into the change due to the income effect and the change due to the substitution effect. You should be able to give exact numerical values.

After the price change, the new optimal number of hats will be:

$$H = \frac{I}{2p_H} = \frac{400}{2 \cdot 10} = 20$$

To decompose the change in hats into the portion due to the income effect and the portion due to the substitution effect, it is necessary to find the intermediate bundle, the bundle corresponding to the new prices but with income adjusted to make the original bundle just affordable. This new bundle requires knowing the original number of shirts purchased. To get this, we need the demand function for shirts. We can get this function by substituting the demand function for H into the result from the tangency condition:

$$S = \frac{p_H}{p_S} H$$

$$S = \frac{p_H}{p_S} \frac{I}{2p_H}$$

$$S = \frac{I}{2p_S}$$

So, plugging in the price of \$10 for shirts and the income of \$400, the original quantity of shirts would have also been 20. Now we can find the amount of income required to purchase the original bundle under the new prices:

$$\tilde{I} = p_H H + p'_S S$$

$$\tilde{I} = 10 \cdot 20 + 5 \cdot 20$$

$$\tilde{I} = 300$$

Now we can find the number of hats in the intermediate bundle using this adjusted income and the new prices:

$$H = \frac{300}{2 \cdot 10} = 15$$

Going from the initial bundle to the intermediate bundle, the number of hats has decreased by 5. This is the change due to the substitution effect. Going from the intermediate bundle to the final bundle, the number of hats has increased by 5. This is the change due to the income effect.

4. (25 points) Suppose that you have 20 hours to study for two midterms, one in economics and one in calculus. If you study for a total of H_E hours for economics, your midterm score in economics (S_E) will be:

$$S_E(H_E) = 100 - \frac{20}{H_E} \tag{4}$$

If you study for a total of H_C hours for calculus, your midterm score in calculus (S_H) will be:

$$S_C(H_C) = 100 - \frac{40}{H_C} \tag{5}$$

(a) Suppose that you care about your average exam score between the two classes. Write down a utility function that captures your preferences over combinations of H_E and H_C.

The simplest approach is to simply say utility is equal to the average exam score:

$$U(H_E, H_C) = \frac{S_E(H_E) + S_C(H_C)}{2}$$

$$U(H_E, H_C) = \frac{100 - \frac{20}{H_E} + 100 - \frac{40}{H_C}}{2}$$

$$U(H_E, H_C) = 100 - \frac{10}{H_E} - \frac{20}{H_C}$$

(b) Given your answer to (a), are your preferences convex? Be certain to fully justify your answer.

Notice that the marginal utility of H_E and the marginal utility of H_C are both diminishing:

$$MU_{H_E} = \frac{dU}{dH_E} = \frac{10}{H_E^2}$$

$$MU_{H_C} = \frac{dU}{dH_C} = \frac{20}{H_C^2}$$

As H_E increases, MU_{H_E} decreases and as H_C increases, MU_{H_E} decreases. The marginal rate of substitution, on a graph with H_E on the horizontal axis, will be:

$$MRS = -\frac{MU_{H_E}}{MU_{H_C}}$$

$$MRS = -\frac{10H_E^{-2}}{20H_C^{-2}}$$

$$MRS = -\frac{C^2}{2E^2}$$

Notice that as you move down and right along an indifference curve (E increasing and C decreasing) the magnitude of the MRS decreases, making the indifference curve flatter and giving us convex preferences.

(c) Suppose that instead of the preferences in part (a), you care about your lowest score. In other words, you are happier the higher your lowest score is. The higher of your two midterm scores does not affect your happiness. Write down a utility function that captures these preferences over combinations of H_E and H_C.

Now our utility is given by the minimum of the two scores:

$$U(H_E, H_C) = \min(S_E(H_E), S_C(H_C))$$
$$U(H_E, H_C) = \min(100 - \frac{20}{H_E}, 100 - \frac{40}{H_C})$$
$$U(H_E, H_C) = \min(-\frac{20}{H_E}, -\frac{40}{H_C})$$
$$U(H_E, H_C) = \max(\frac{20}{H_E}, \frac{40}{H_C})$$
$$U(H_E, H_C) = \min(\frac{H_E}{20}, \frac{H_C}{40})$$
$$U(H_E, H_C) = 40\min(2H_E, H_C)$$

(d) On a graph with hours of studying for economics on the horizontal axis and hours of studying for calculus on the vertical axis, show your budget line and the indifference curve passing through your optimal bundle (using the preferences from part (c)). Be certain to label the optimal bundle and any relevant intercepts with their numerical values.

First let's find the optimal bundle. Notice that if one argument of the *min* function above is larger than the other, we can increase utility by shifting time from the subject with the larger argument to the subject with the smaller argument. Therefore, utility will only be maximized when the two arguments are equal:

$$2H_E = H_C$$

Our budget constraint is that the combination of H_E and H_C can be no larger than 20 hours:

$$H_E + H_C = 20$$
$$H_E + 2H_E = 20$$
$$3H_E = 20$$
$$H_E = \frac{20}{3}$$
$$H_C = \frac{40}{3}$$

So your graph should have a budget line going from 20 hours of chem and zero hours of econ studying to zero hours of chem studying and 20 hours of econ studying. The optimal bundle should be on this budget line at $\frac{20}{3}$ hours of econ and $\frac{40}{3}$ hours of chem. The indifference curve should be L-shaped with the the vertex at this optimal bundle.

5. (20 points) Suppose that you are always will to trade two cans of Sprite for three cans of ginger ale. You currently have \$20 to spend on Sprite and ginger ale as well as five coupons for a free can of Sprite (each coupon gets you one free can of Sprite).

   (a) On a graph with cans of Sprite (S) on the horizontal axis and cans of ginger ale (G) on the vertical axis, show your set of affordable bundles of Sprite and ginger ale if the price of a can of Sprite is \$1 and the price of a can of ginger ale is \$1.

   Your budget line should have a horizontal segment going from the point with 20 cans of ginger ale and zero cans of Sprite to 20 cans of ginger ale and 5 cans of Sprite (this is the point corresponding to spending all of your money on ginger ale and using your coupons for Sprite). The rest of your budget line should be a straight line extending from 20 cans of ginger ale and 5 cans of Sprite to zero cans of ginger ale and 25 cans of Sprite (the point corresponding to spending all of your money on 20 cans of Sprite and using your coupons to get 5 more cans).

   (b) On the same graph, show at least one indifference curve. Label the slope of the indifference curve with its numerical value.

   Your indifference curve should be a straight line with a slope of $-\frac{3}{2}$ (the slope of the indifference curve is equal to the number of cans of ginger ale you are willing to trade for the a can of Sprite and you're willing to trade three ginger ale cans for two Sprite cans).

   (c) What is the utility maximizing combination of Sprite and ginger ale?

   Note that the ratio of the price of Sprite to the price of ginger ale is less than the ratio of the marginal utility of Sprite to the marginal utility of ginger ale (the slope of the indifference curve). So ginger ale costs more relative to Sprite than it adds to utility relative to Sprite. Sprite is the better deal. So you will spend all of your money on Sprite and use the coupons for Sprite, leading to an optimal bundle of 25 cans of Sprite and no cans of ginger ale.

   (d) Now suppose that the price of ginger ale decreases to \$0.50 a can. What is your new optimal bundle?

   Now the price of Sprite relative to the price of ginger ale (two to one) is greater than the marginal utility of Sprite relative to the marginal utility of ginger ale. So ginger ale is now a better deal than Sprite. You will use all of your money on ginger ale to get 20 cans of ginger ale and then use your five coupons for five cans of Sprite (you get no utility from not using these coupons). You're optimal bundle is now at the kink in the budget line.