Econ 303 - Intermediate Microeconomic Theory
College of William and Mary May 5, 2014
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Final Exam - Solutions

You have until 3:30pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can use fractions of units of inputs and produce fractions of units of output. You may also assume that consumers can buy and consume fractions of units of goods. Remember to put your name on the exam. Good luck!

Name:

1. (15 points) Suppose that a cable company has to invest a large amount in fiberoptic cables to set up Williamsburg for cable. The size of this investment is independent of how many cable subscribers the company actually has. Once the fiberoptic cables are set up, each additional customer adds $10 to the company's total costs regardless of how many customers are currently subscribing to cable. Demand for cable decreases as price increases, falling by a constant number of customers for each additional dollar in price (so the demand curve is linear). On a graph with number of cable customers on the horizontal axis, graph the following items:

   (a) The average total cost curve and marginal cost curve for the cable company.
   (b) The profits the cable company will make if allowed to act as a monopolist but not allowed to price discriminate.
   (c) The profits the cable company will make if forced to serve the efficient number of customers.
   (d) The price at which the cable company would break even.

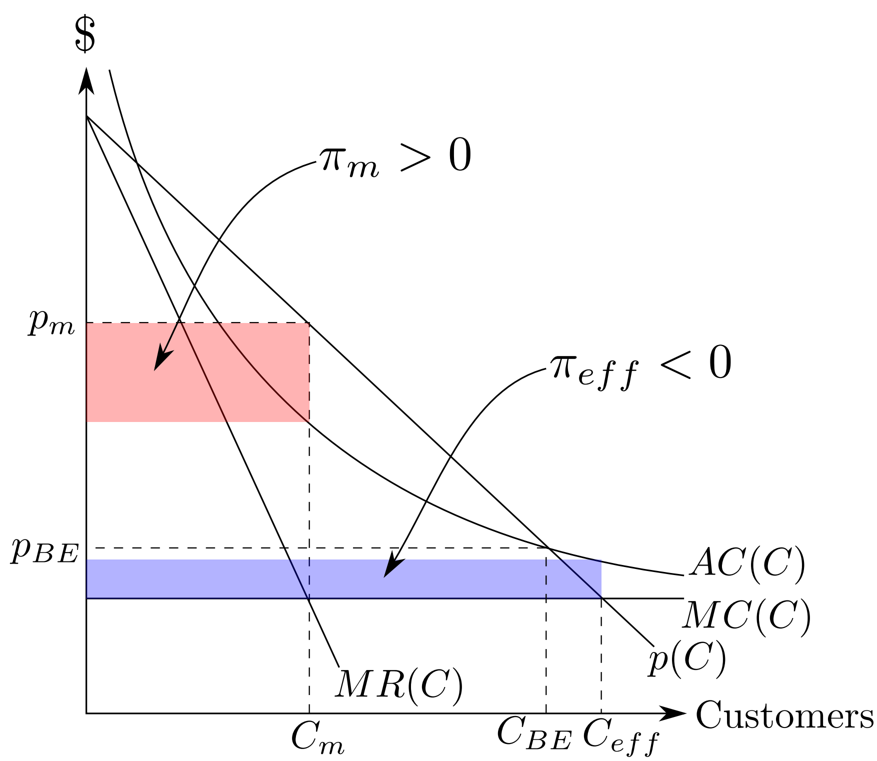
   Be certain to clearly label all components of your graph. Also clearly note whether profits are positive or negative for parts (b) and (c).

   The fact that each additional customer adds $10 to the company's total costs regardless of the number of current customers tells us that marginal costs and average variable costs are constant and equal to $10. So the marginal cost curve and the average variable cost curve are both a horizontal line at $10. The average total cost curve will start out high because the high fixed costs of the fiberoptic cable will be spread across a small number of units. As the number of customers increases, these fixed costs will be spread across more and more customers lower the average fixed costs and leading to average total costs approaching the average variable costs line.

   When acting as a monopolist, the firm will produce up to the point where marginal revenue equals marginal cost. For a monopolist, the marginal revenue curve will have the same intercept as the demand curve but a slope that is twice as steep. The monopolist will charge whatever consumers are willing to pay at that quantity. This quantity and price are labeled as C_m and p_m on the graph. The profits at this price and quantity can be found by comparing price to average cost at that quantity, giving the profit per unit, and multiplying by the number of units. These profits are shown as the area π_m on the graph and are positive. *Note that if you drew your graph with the average cost curve above the demand curve at all quantities, the profits at C_m will be negative and the firm would actually choose not to produce at all.*

The efficient quantity will be where the marginal cost curve intersects the demand curve. This quantity is labeled C_{eff} on the graph. You would find profits at this point the same way we found profits at the monopoly price and quantity. The profits are labeled as π_{eff} on the graph. Note that profits are negative at this point because the average costs are above the price.

The break-even price, labeled p_{BE} on the graph, will be where price is just equal to average costs. At this point, the firm is earning zero profits (breaking even) on each unit.



2. (25 points) Suppose that you can enter the local DVD rental business by either opening a store or by setting up a kiosk (like Redbox). The total costs of running a DVD store are given by:

$$C_S(D) = D^2 + 2D \tag{1}$$

where D is the number of DVDs rented. The total costs of running a kiosk are given by:

$$C_K(D) = 2D^2 + D \tag{2}$$

where once again D is the number of DVDs rented. Demand for DVDs in Williamsburg is given by the following inverse demand function:

$$p(D) = 5 - \frac{1}{100}D \tag{3}$$

where $p(D)$ is the price of a DVD rental and D is the number of rentals.

(a) Find expressions for the average total costs ($AC_S(D)$), average variable costs ($AVC_S(D)$), and marginal costs ($MC_S(D)$) of running a store and the average total costs ($AC_K(D)$), average variable costs ($AVC_K(D)$), and marginal costs ($MC_K(D)$) of running a kiosk.

First note that for both types of businesses, all costs are variable (D is in every term). So average total costs and average variable costs will both simply be the total cost function divided by quantity. Marginal costs are the derivative of the cost function with respect to D (telling us how costs change when D changes by a small amount).

$$AC_S(D) = \frac{C_S(D)}{D} = \frac{D^2 + 2D}{D} = D + 2$$

$$AVC_S(D) = \frac{VC_S(D)}{D} = \frac{D^2 + 2D}{D} = D + 2$$

$$MC_S(D) = \frac{dC_S(D)}{dD} = 2D + 2$$

$$AC_K(D) = \frac{C_K(D)}{D} = \frac{2D^2 + D}{D} = 2D + 1$$

$$AVC_K(D) = \frac{VC_K(D)}{D} = \frac{2D^2 + D}{D} = 2D + 1$$

$$MC_K(D) = \frac{dC_K(D)}{dD} = 4D + 1$$

(b) Assuming that the DVD rental market is highly competitive and firms can enter and leave easily, which type of rental service will exist in the long run, stores or kiosks? Be certain to fully justify your answer.

In the long run, price will be driven down to the break-even price. However, if the two types of business have different break-even prices, the one with the higher break-even price will be driven out of the market by the businesses with the lower break-even price. So we need to figure out the type of business with the lowest break-even price. This will be the type that remains in business in the long run. To find the break-even price, we need to find the quantity at which the marginal cost curve intersects the average cost curve and then see what price this quantity corresponds to by plugging the quantity back into the marginal cost function. Let's start with the store:

$$MC_S(D_{BE}) = AC_S(D_{BE})$$

$$2D_{BE} + 2 = D_{BE} + 2$$

$$2D_{BE} = D_{BE}$$

The only value of D_{BE} that solves this is zero, so our quantity at the break-even price is zero. To find the corresponding price we now plug this quantity back into the marginal cost function:

$$p_S^{BE} = MC_S(0)$$

$$p_S^{BE} = 2 \cdot 0 + 2 = 2$$

So at a price of \$2, a rental store just breaks even. If price is driven below \$2, rental stores will exit the market. Now we need to repeat this procedure for the rental kiosks:

$$MC_K(D_{BE}) = AC_K(D_{BE})$$

$$4D_{BE} + 1 = 2D_{BE} + 1$$

$$4D_{BE} = 2D_{BE}$$

$$D_{BE} = 0$$

$$p_K^{BE} = MC_K(0)$$

$$p_K^{BE} = 4 \cdot 0 + 1 = 1$$

Kiosks will stay in business all the way down to a rental price of \$1. So in the long run, stores will shut down and only kiosks will remain as competition drives price down to \$1.

(c) What will the long run equilibrium price of a DVD rental be and how many DVDs will be rented in the long run?

From above we found that the lowest break-even price will be \$1. In the long run, firms will be earning zero profits (otherwise we wouldn't be in equilibrium because firms would want to enter or leave), so price will be equal to this break-even price of \$1 (note that the break-even price of \$2 for stores is not relevant because at that price, kiosks are still earning positive profits and would still be entering the industry pushing price down and the number of rentals up). The equilibrium quantity will be whatever customers demand at this price:

$$p(D) = 5 - \frac{1}{100}D$$

$$1 = 5 - \frac{1}{100}D$$
$$\frac{1}{100}D = 4$$
$$D = 400$$

(d) Suppose that Williamsburg imposes a \$1 quantity tax on DVD rentals. Once this tax is in place, what will the long run quantity of DVD rentals be, what price will consumers pay for a rental and what price will firms receive for a rental?

Firms still need to earn zero profits in our long run equilibrium. So the price received by firms will still need to be \$1 in the long run. However, this means the consumers will need to be paying that \$1 plus the tax of \$1, making the total price paid by consumers \$2. At a price of \$2, the quantity of rentals will be:

$$p(D) = 5 - \frac{1}{100}D$$
$$2 = 5 - \frac{1}{100}D$$
$$\frac{1}{100}D = 3$$
$$D = 300$$

So the tax has reduced the equilibrium quantity to 300 rentals.

3. (25 points) There are two bookstores in Williamsburg. Bookstore A has no fixed costs and constant marginal costs of \$5 per book. Bookstore B also has no fixed costs and but has higher marginal costs of \$6 per book due to a bad distribution deal with the publishing company. Demand for books in Williamsburg is given by:

$$D(p) = 100 - 10p \tag{4}$$

The two bookstores set their orders for books at the beginning of each month. The market price for books in Williamsburg that month is then determined by the total number of books supplied by the two bookstores combined.

(a) Write down an equation giving the profits for bookstore A as a function of the number of books it orders (B_A) and the number of books bookstore B orders (B_B).

Profits will simply be revenues minus costs. The revenue will depend both on B_A (because B_A affects price and because B_A is the number of books the store gets revenue on) and on B_B (because B_B affects the price of a book). The costs will simply depend on B_A. Since there are no fixed costs and each book has a marginal cost of \$5, total costs will simply be $5B_A$. To get revenues in terms of B_A and B_B we first need to get price in terms of B_A and B_B by rearranging the demand function to get an inverse demand function:

$$\begin{aligned} D(p) &= 100 - 10p \\ B_A + B_B &= 100 - 10p(B_A, B_B) \\ 10p(B_A, B_B) &= 100 - B_A - B_B \\ p(B_A, B_B) &= 10 - \frac{1}{10}B_A - \frac{1}{10}B_B \end{aligned}$$

Now we can write out our profit function:

$$\pi_A(B_A, B_B) = p(B_A, B_B) \cdot B_A - C_A(B_A)$$

$$\pi_A(B_A, B_B) = \left(10 - \frac{1}{10}B_A - \frac{1}{10}B\right) \cdot B_A - 5B_A$$

(b) Find an expression for bookstore A's marginal revenue as a function of B_A and B_B. Explain why this marginal revenue function is decreasing as B_A increasing.

From our profit function above, the revenue of bookstore A is given by:

$$R_A(B_A, B_B) = \left(10 - \frac{1}{10}B_A - \frac{1}{10}B_B\right) \cdot B_A$$

Expanding this out gives us:

$$R_A(B_A, B_B) = 10B_A - \frac{1}{10}B_A^2 - \frac{1}{10}B_A B_B$$

The marginal revenue for bookstore A is simply the derivative of this revenue function with respect to B_A:

$$MR_A(B_A, B_B) = \frac{dR_A}{dB_A}$$

$$MR_A(B_A, B_B) = 10 - \frac{1}{5}B_A - \frac{1}{10}B_B$$

Notice that as B_A increases, this marginal revenue function decreases. There are two effects on revenue from an increase in B_A. The first is that revenue increases from selling one extra unit. The second is that revenue decreases because the firm loses revenue on the previous units from lowering price in order to sell the additional unit. The first effect on revenue is getting smaller as B_A gets bigger because the price of a book (which is the added revenue from the next book) is getting lower as B_A increases. The second component is getting larger as B_A increases since there are more and more books affected by the price cut. So the positive component of the marginal revenue is getting smaller and the negative component is getting larger as B_A gets bigger leading to declining marginal revenue overall as B_A is increased.

(c) Suppose that bookstore B orders 20 books. What is the profit-maximizing number of books for firm A to order?

The bookstore will maximize profits by selling up to the point where marginal revenue equals marginal cost:

$$MR_A(B_A, B_B) = MC_A(B_A)$$

Plugging in our functions for marginal revenue and marginal cost and then solving for B_A gives us:

$$10 - \frac{1}{5}B_A - \frac{1}{10}B_B = 5$$

$$\frac{1}{5}B_A = 5 - \frac{1}{10}B_B$$

$$B_A = 25 - \frac{1}{2}B_B$$

This is bookstore A's best response function. It gives the optimal quantity of books to order given a particular number of books ordered by bookstore B. Plugging in a value of 20 books for B_B into this function gives us 15 as the optimal number of books for bookstore A to order.

(d) Suppose that bookstore A orders 20 books. What is the profit-maximizing number of books for firm B to order?

We need to go through the same process as we did for bookstore A to find bookstore B's best response function, setting marginal revenue equal to marginal cost and solving for B_B. However, we can take one shortcut. Notice that the revenue function for bookstore B will look just like the revenue function for bookstore A only with B_B's instead of the B_A's and B_A's instead of the B_B's. On the marginal cost side, bookstore B has slightly higher marginal costs of \$6. With this information, we can now solve for bookstore B's best response function:

$$MR_B(B_B, B_A) = MC_B(B_B)$$

$$10 - \frac{1}{5}B_B - \frac{1}{10}B_A = 6$$
$$\frac{1}{5}B_B = 4 - \frac{1}{10}B_A$$
$$B_B = 20 - \frac{1}{2}B_A$$

Plugging in 20 for B_A in this equation gives us 10 as bookstore B's optimal quantity.

(e) At what quantity of B_B would bookstore A decide to order no books at all? Why would you not expect to see this happen? In other words, why should it never be the case the bookstore B sells books but bookstore A does not? Make certain your answer takes into consideration the best response functions of the two firms.

Using our best response function for bookstore A from above we can plug in zero for B_A and solve for B_B:

$$0 = 25 - \frac{1}{2}B_B$$
$$\frac{1}{2}B_B = 25$$
$$B_B = 50$$

So when bookstore B is ordering 50 books, bookstore A's optimal decision would be to drop out of the market. However, this is not a very likely scenario. Notice that no matter what bookstore B thinks bookstore A is doing, bookstore B would never choose a quantity greater than 20 based on its best response function (the best response function gives $B_B \leq 0$ for all values of $B_A \geq 0$). So we would not expect to see bookstore B ever order enough books to convince bookstore A to drop out of the market. Furthermore, note that at a quantity of 50 books, consumers would be willing to pay \$5 a book which is below bookstore B's cost per book. Bookstore B would be losing money even if bookstore A were selling no books (any additional books sold by bookstore A would drive price even further below bookstore B's cost per book).

4. (20 points) The College plants flowers on campus to attract prospective students. For every additional flower planted, the College gains \$100 in expected profits from future students. Current students also benefit from the flowers because they look nice. The marginal benefit current students get from additional flowers decreases as the number of flowers increases since the students start to get desensitized to the beauty of the flowers. The College does not care about the marginal benefits to current students since current students are already committed to paying their tuition. Suppose that the marginal costs to the college of planting flowers are given by:

$$MC(F) = 5F \tag{5}$$

where F is the current number of flowers planted.

(a) On a graph with number of flowers on the horizontal axis, show the marginal costs to the College of planting flowers, the marginal benefits to the College of planting flowers, and the social marginal benefits of planting flowers.

The marginal cost curve is given by the equation above and should be an upward sloping line with a slope of 5 passing through the origin. The marginal benefits for the College are constant and equal to \$100 so the marginal benefit curve for the College should be a horizontal line at \$100. The social marginal benefit curve includes these constant marginal benefits to the College as well as the marginal benefit to current students. Since these marginal benefits to current students decrease as F increases, the social marginal curve should be downward sloping and approach the College marginal benefit curve as F gets larger.

(b) On your graph, show the quantity of flowers that the College will decide to plant.

The College will only plant flowers for which the College's marginal benefit exceeds the College's marginal cost. So the College will plant up to the point where its marginal benefit curve intersects its marginal cost curve. This quantity of flowers is labeled F_C on the graph.

(c) On your graph, show the socially efficient quantity of flowers.

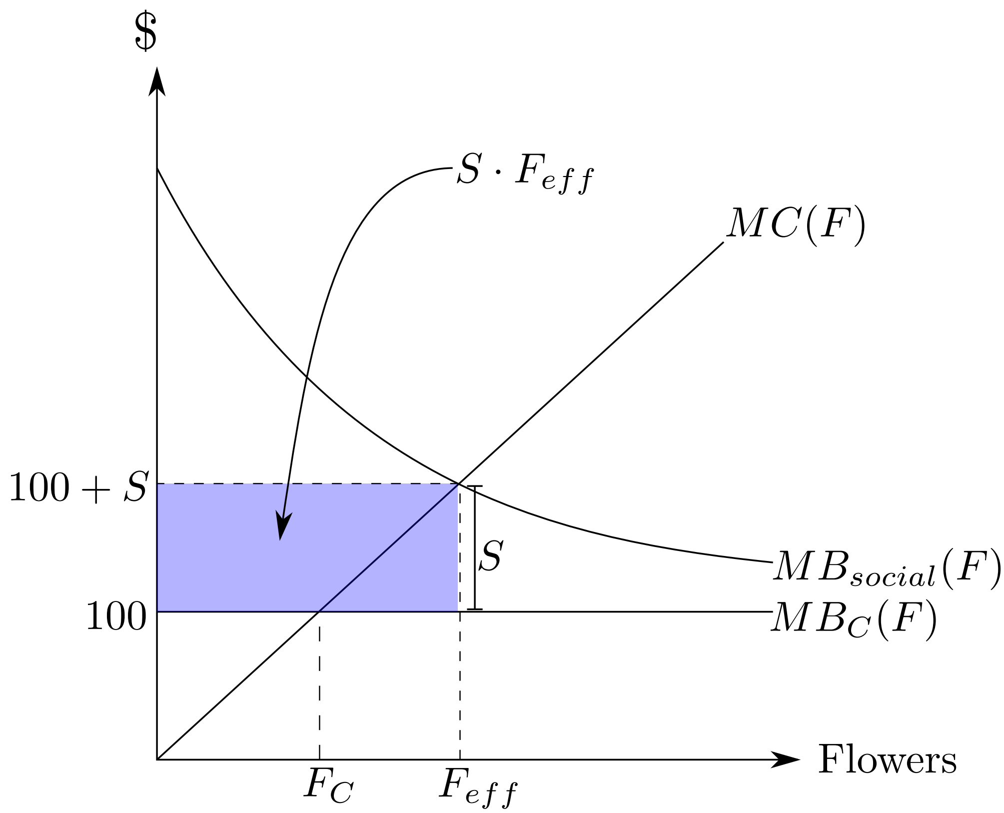
It is socially efficient to plant any flowers for which the social marginal benefit exceeds the social marginal cost. So the socially efficient quantity of flowers is where the marginal cost curve intersects the social marginal benefit curve. This quantity of flowers is labeled F_{eff} on the graph.

(d) The City of Williamsburg wants to get the college to plant the socially efficient quantity of flowers. The City plans to offer a subsidy to the college per flower planted. On your graph, show the size of the subsidy per flower the City should offer and the total amount the City will have to pay in subsidies.

The subsidy will effectively shift the College's marginal benefit curve up by the amount of the subsidy per flower. The City wants to shift the College's curve enough so that the College chooses to plant F_{eff} flowers. In other words, the subsidy needs to shift the College's marginal benefit curve up until it intersects the marginal cost curve at F_{eff}. The size of the subsidy that will accomplish this is equal to the gap between the College's marginal benefit curve and the social marginal benefit curve at F_{eff}. The total amount spent on subsidies will be equal to the subsidy per flower, S on the graph, times the number of flowers planted, F_{eff}. This is the shaded area on the graph.

(e) If the subsidy above is implemented, will total surplus increase or decrease? Be certain to fully explain your answer and consider all relevant components of total surplus.

Total surplus will increase. The total surplus on the flowers up to F_C remains the same. The flowers from F_C to F_{eff} all add to total surplus (social marginal benefit exceeds social marginal cost on all of them). Even though the subsidies are a loss to total surplus (they are a cost to the taxpayer that did not exist before the subsidy was put in place), that loss is outweighed by the gain in surplus for the college from getting the subsidies and the gain in surplus for current students from more flowers being planted.



5. (15 points) There is only one berry farm in the Hampton Roads area so the farmer can act as a monopolist. The total costs of growing and distributing B baskets of berries are:

$$C(B) = 10 + B^2 \tag{6}$$

The inverse demand curve for berries is given by:

$$p(B) = 60 - \frac{1}{2}B \tag{7}$$

where $p(B)$ is the price per basket a customer is willing to pay for B baskets of berries.

(a) Suppose that the farmer decides to set a single price per basket. What would the profit-maximizing price be and what profits would the farmer make?

To maximize profits, the farmer will sell baskets up to the point where marginal revenue is equal to marginal cost. Marginal revenue can be found by writing out the revenue function in terms of B and then taking the derivative with respect to B:

$$R(B) = p(B) \cdot B$$

$$R(B) = (60 - \frac{1}{2}B)B$$

$$R(B) = 60B - \frac{1}{2}B^2$$

$$MR(B) = \frac{dR(B)}{dB}$$

$$MR(B) = 60 - B$$

Marginal cost is given by the derivative of the total cost function with respect to B:

$$MC(B) = \frac{dC(B)}{dB}$$

$$MC(B) = 2B$$

Now we can set marginal revenue equal to marginal cost:

$$MR(B) = MC(B)$$

$$60 - B = 2B$$

$$3B = 60$$

$$B = 20$$

The price that consumers are willing to pay at this quantity will be:

$$p(20) = 60 - \frac{1}{2} \cdot 20$$

$$p(20) = 50$$

Total profits at this price and quantity will be:

$$\pi = p(B) \cdot B - C(B)$$

$$\pi = 50 \cdot 20 - 10 - 20^2$$

$$\pi = 590$$

(b) Now suppose that the farmer instead decides to sell berries through a membership plan. Customers must pay a one-time membership fee of M and can then buy as many baskets as they want at a price p. What are the profit-maximizing membership fee and price per basket? What will the farmer's total profits be?

This is an example of a two part tariff. With this pricing structure, the farmer should sell every basket for which marginal benefit exceeds marginal cost because he can capture all of that net surplus either through the price per basket or through the membership fee. So the farmer will sell up to the point where the marginal cost curve intersects the demand curve:

$$MC(B) = p(B)$$

$$2B = 60 - \frac{1}{2}B$$

$$\frac{5}{2}B = 60$$

$$B = 24$$

So the farmer will sell 24 baskets. The price per basket that corresponds to this quantity is:

$$p(24) = 60 - \frac{1}{2} \cdot 24$$

$$p(24) = 48$$

As for the membership fee, the farmer can charge customers any amount up to their consumer surplus at a quantity of 24 baskets and a price of \$48 per basket and they will take the deal (if they do not take the deal they are left with a surplus of zero). The consumer surplus at 24 baskets and \$48 a basket is given by the area under the demand curve above the price of \$48 up to the quantity of 24 baskets:

$$CS = \frac{1}{2}(p(0) - p) \cdot B = \frac{1}{2}(60 - 48) \cdot 24 = 144$$

So the farmer will charge \$144 for the membership fee. The farmer's profits will be:

$$\pi = p(B) \cdot B - C(B) + \text{fee}$$

$$\pi = 48 \cdot 24 - 10 - 24^2 + 144$$

$$\pi = 710$$

(c) Now the farmer decides to offer both plans, the membership plan from part (b) (at the membership fee and price per basket you found in part (b)) and the non-membership approach (no fee and the price per basket you found in part (a)). Which plan will the customer take? Be certain to fully justify your answer.

The customer faces a choice between a price of \$50 or a price of \$48 with a membership fee of \$144. Recall that the membership was set equal to the customer's consumer surplus at a price of \$48 per basket. Therefore after paying the membership fee the customer is left with zero consumer surplus under the membership plan approach. At the higher basket price of \$50, the consumer is paying more per basket but does not have to pay a membership fee. The consumer will get to keep his consumer surplus at this price of \$50. Given that the consumer is left with positive surplus under the \$50 price and zero surplus under the membership plan, the customer will opt for the \$50 price.