Econ 303 - Intermediate Microeconomic Theory
College of William and Mary — December 16, 2013
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Final Exam - Solutions

You have until 3:30pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Calculators may be used although you may leave answers as fractions. Unless a problem says otherwise, you can assume that individuals can consume fractions of units and firms can use fractions of units of inputs and produce fractions of units of output. Remember to put your name on the exam. Good luck!

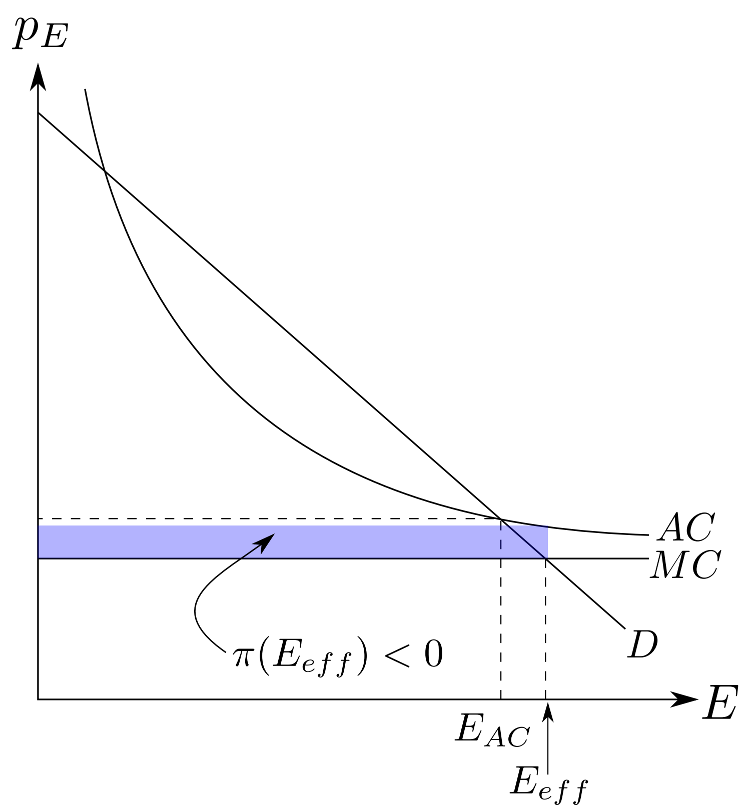
Name:

1. (20 points) Suppose that a local electricity provider has very high fixed costs and constant marginal costs for providing electricity. The demand curve for electricity is linear and downward sloping.

   (a) Explain why for any given quantity of electricity it is socially efficient to have a single electricity provider rather than multiple electricity providers.

   > Note that the fixed costs and constant marginal costs imply a downward sloping average cost curve (average variable costs are staying constant due to the constant marginal costs but average fixed costs fall as quantity increases due to the fixed costs be spread across more and more units). If we split production of a total quantity y evenly across n firms, the average costs at each firm would be $AC(\frac{y}{n})$. Because this is a decreasing function of quantity, it will increase as n increases. So n equal to one leads to the smallest possible average costs per firm. For a given quantity, and therefore a given gross benefit to consumers, one firm leads to the lowest costs, maximizing total surplus.

   (b) Use a clearly labeled graph to determine whether the electricity provider would lose money if forced to provide the socially efficient level of electricity. You can assume that the local government can dictate the quantity of electricity but that the firm can then charge whatever price consumers are willing to pay at that quantity.



The graph above shows a linear, downward sloping demand curve, a horizontal marginal cost curve (due to the constant marginal costs) and a downward sloping average cost curve that approaches the marginal cost curve as quantity increases (spreading the fixed costs across more and more units). The socially efficient quantity, E_{eff} is where the marginal cost curve intersects the demand curve. For every unit of electricity up to this point, the marginal benefits (shown by the demand curve) exceed the marginal costs making it socially efficient to produce that electricity. For every unit after that, the marginal costs exceed the marginal benefits, so the units would reduce total surplus if produced. At this quantity, average costs exceed the price the firm can charge so the firm would lose money on each unit. The total losses are shown as the shaded area on the graph (the difference between price and average cost multiplied by the number of units).

(c) On the same graph, show the largest quantity of electricity the local government could force the firm to provide before the firm decides to shut down.

As long as the price consumers are willing to pay is above the average cost of the firm, the firm can earn positive profits and would be willing to stay in business. This will be the case up to the point where the average cost curve intersects the demand curve. The quantity at which this occurs is labeled E_{AC} on the graph. Beyond this quantity the firm would lose money and would prefer to shut down.

(d) How would your answers to (a) and (b) change if demand for electricity was perfectly inelastic?

If demand were perfectly inelastic, customers would always demand some particular quantity of electricity E^* regardless of the price. So the socially efficient quantity would now be E^* (consumers have essentially an infinite marginal benefit on the units before that and zero marginal benefit on units after that). The firm would be willing to provide E^* because they can charge the consumers enough to cover average costs and earn positive profits (consumers are willing to pay any price at E^* including prices substantially higher than the average costs). So the firm will be willing to supply the efficient quantity and will earn positive profits at that quantity.

2. (20 points) There are two farmers growing corn, McDonald and Fannie. McDonald has good soil for growing corn. For every additional bushel he grows, his total costs increase by \$25. Fannie has bad soil, her costs rise by \$50 for every additional bushel of corn she grows. For simplicity, assume that each farmer has no fixed costs. The demand for corn is given by the following demand function:

$$D(p) = 900 - 9p \tag{1}$$

where p is the price of a bushel of corn. The two farmers compete by choosing quantities. The market price is then determined by what consumers are willing to pay for the combined output of the farmers.

(a) Write down an equation for McDonald's profits (π_M) as a function of the bushels of corn grown by McDonald (B_M) and the bushels of corn grown by Fannie (B_F). Your equation should contain no parameters or variables other than B_M and B_F.

McDonald's profits will be his revenues minus his costs. His revenues will depend on both the quantity he decides to sell and the market price which will be a function of both his quantity and Fannie's quantity. His costs will just be a function of his own chosen quantity:

$$\pi_M(B_M, B_F) = p(B_M + B_F) \cdot B_M - C_M(B_M)$$

Now we need to get the inverse demand function by rearranging the demand function given above:

$$B_{total} = 900 - 9p(B_{total})$$
$$9p(B_{total}) = 900 - B_{total}$$
$$p(B_{total}) = 100 - \frac{1}{9}B_{total}$$

Plugging this in for the inverse demand function in the profit equation and noting that the constant marginal costs per unit of \$25 implies $C_M(B_M) = 25B_M$ gives us:

$$\pi_M(B_M, B_F) = (100 - \frac{1}{9}(B_M + B_F)) \cdot B_M - 25B_M$$
$$\pi_M(B_M, B_F) = 100B_M - \frac{1}{9}B_M^2 - \frac{1}{9}B_F B_M - 25B_M$$
$$\pi_M(B_M, B_F) = 75B_M - \frac{1}{9}B_M^2 - \frac{1}{9}B_F B_M$$

(b) Given your equation in part (a), derive an expression giving McDonald's optimal quantity of corn B_M in response to Fannie's chosen quantity of corn B_F (you are deriving McDonald's best response function $B_M(B_F)$).

McDonald will choose the quantity of corn B_M that maximizes the above profit function, treating B_F as given. To find this value of B_M, we can take the derivative of the profit function with respect to B_M and set it equal to zero:

$$\frac{d\pi_M}{dB_M} = 75 - \frac{2}{9}B_M - \frac{1}{9}B_F$$

$$0 = 75 - \frac{2}{9}B_M - \frac{1}{9}B_F$$
$$\frac{2}{9}B_M = 75 - \frac{1}{9}B_F$$
$$B_M = \frac{675}{2} - \frac{1}{2}B_F$$

This is McDonald's best response to any particular quantity provided by Fannie.

(c) Derive Fannie's best response function, $B_F(B_M)$, giving Fannie's optimal quantity of corn as a function of the amount of corn grown by McDonald.

We can follow the same approach we did for McDonald. The only difference is that Fannie has a different cost function ($50B_F$ as opposed to $25B_M$). We start by first writing out Fannie's profit function:

$$\pi_F(B_F, B_M) = (100 - \frac{1}{9}(B_M + B_F)) \cdot B_F - 50B_F$$
$$\pi_F(B_F, B_M) = 100B_F - \frac{1}{9}B_F^2 - \frac{1}{9}B_M B_F - 50B_F$$
$$\pi_F(B_F, B_M) = 50B_F - \frac{1}{9}B_F^2 - \frac{1}{9}B_M B_F$$

Now we take the derivative with respect to B_F and set it equal to zero to find Fannie's optimal quantity:

$$\frac{d\pi_F}{dB_F} = 50 - \frac{2}{9}B_F - \frac{1}{9}B_M$$
$$0 = 50 - \frac{2}{9}B_F - \frac{1}{9}B_M$$
$$\frac{2}{9}B_F = 50 - \frac{1}{9}B_M$$
$$B_F = 225 - \frac{1}{2}B_M$$

This is Fannie's best response to any particular quantity provided by McDonald.

(d) Find the equilibrium quantity of corn grown by McDonald and the equilibrium quantity of corn grown by Fannie.

The equilibrium quantities will be the values of B_M and B_F that solve both of the best response functions. At these values, McDonald does not want to change B_M given Fannie's response to B_M and Fannie does not want to change B_F given McDonald's response to B_F. So both farmers are content to stick to their current quantities. To find the values of B_M and B_F that solve both best response functions, we can substitute one equation into the other:

$$B_M = \frac{675}{2} - \frac{1}{2}B_F$$
$$B_M = \frac{675}{2} - \frac{1}{2}(225 - \frac{1}{2}B_M)$$

$$B_M = \frac{675}{2} - \frac{225}{2} + \frac{1}{4}B_M$$
$$\frac{3}{4}B_M = 225$$
$$B_M = 300$$

Plugging this back into Fannie's best response function will give us B_F:

$$B_F = 225 - \frac{1}{2}B_M$$

$$B_F = 225 - \frac{1}{2} \cdot 300$$
$$B_F = 75$$

So in equilibrium, McDonald will grow 300 bushels of corn and Fannie will grow 75 bushels of corn. Notice that the farmer with the lower marginal costs ends up growing more corn in equilibrium but doesn't completely drive the other farmer out of the market (which would happen if they competed on price rather than quantity).

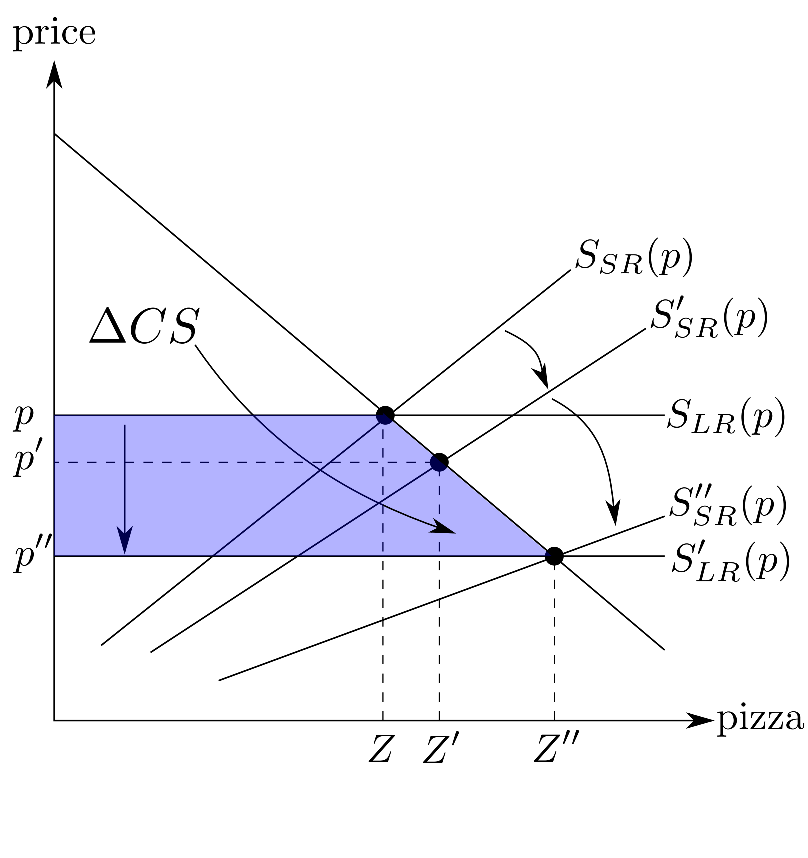
3. (20 points) Consider a perfectly competitive market for pizza that is currently in long run equilibrium. All of the firms in this market are identical and have average costs that are a decreasing function of quantity at low quantities but an increasing function of quantity at larger quantities (in other words, a U-shaped average cost curve). The marginal cost curve for each firm is upward sloping. Suppose that an improvement in pizza oven technology shifts the average costs for all of these firms down at all quantities but does not change the quantity at which the average cost curve reaches its minimum. Use a graph of the market demand curve, short run industry supply curve and long run industry supply curve to show the effects of this technological change on the following:

   (a) The equilibrium price and quantity of pizza in the short run.

   (b) The equilibrium price and quantity of pizza in the long run.

   (c) Consumer surplus in the long run.

   Clearly label all relevant curves and points on your graph and provide a written explanation of each change you show on the graph.



The graph above shows all of the relevant changes. The initial short run and long run supply curves are $S_{SR}(p)$ and $S_{LR}(p)$, respectively. We are initially in long run equilibrium (the intersection of $S_{SR}(p)$, $S_{LR}(p)$ and $D(p)$). The change in technology will reduce the marginal costs for firms at any given quantity. This will affect both the amount firms are willing to supply at any given price, shifting the short run supply curve to the right, and the price at which the firms will break even, shifting the long run supply curve down.

In the short run, there will be a movement down and right along the demand curve to the point of intersection of the demand curve with the new, shifted short run supply curve $S'_{SR}(p)$. So in the short run, the equilibrium price and quantity will move from (p, Z) to (p', Z').

At this new short run equilibrium, firms are earning positive profits (the price is still above the new breakeven price). Given these positive profits, other firms will decide to enter the industry rotating the short run supply curve out until the new intersection with the demand curve occurs at the new breakeven price (the point where the new long run supply curve intersects the demand curve). So short run supply ultimately rotates out to $S''_{SR}(p)$ and the new long run equilibrium ends up being (p'', Z'').

As for the change in consumer surplus, it has two different components. On the Z units that consumers were buying before, they now get to pay a lower price, so there is a gain in consumer surplus equal to the difference in price $(p - p'')$ multiplied by Z. In addition to this, consumers are also getting surplus on the new units that they purchase (the units between Z and Z''). This new consumer surplus is equal to the area under the demand curve above p'' from Z to Z''. The total change in consumer surplus is shown as the shaded region on the graph.

4. (20 points) A record store is going out of business and needs to sell off its inventory. It is the only record store in town and can therefore act as a monopolist. The store is considering two different strategies for its final weekend in business:

   i. Go with a simple approach of setting a single price per record and keeping the doors open to all customers.

   ii. Have an exclusive event where individuals must pay admission to come to the event but then get lower prices per record.

   The record store currently has a stock of 500 records and will simply throw out any records it doesn't sell over the weekend. For simplicity, assume that there is a single customer with the following demand curve for records:

$$D(p) = 500 - 25p \tag{2}$$

   (a) Determine the price the store will charge, the number of records that will be sold and the total profits the store will make if it takes the first approach.

   This first approach is just standard monopoly pricing. The store will find the quantity at which marginal revenue equals marginal cost and charge the price consumers are willing to pay at this quantity. Let's start by getting the inverse demand function and then using that to find the store's marginal revenue function:

$$\begin{aligned} y &= 500 - 25p(y) \\ 25p(y) &= 500 - y \\ p(y) &= 20 - \frac{1}{25}y \\ R(y) &= p(y) \cdot y \\ R(y) &= (20 - \frac{1}{25}y)y \\ R(y) &= 20y - \frac{1}{25}y^2 \\ MR(y) &= \frac{dR(y)}{dy} = 20 - \frac{2}{25}y \end{aligned}$$

   Now we can set marginal revenue equal to marginal cost to get the profit-maximizing number of records. Note that in this case the marginal cost of selling an additional record is simply zero; the store already has its stock of records and will simply throw out any unsold records.

$$\begin{aligned} MR(y) &= MC(y) \\ 20 - \frac{2}{25}y &= 0 \\ \frac{2}{25}y &= 20 \\ y &= 250 \end{aligned}$$

So the store will sell 250 records. At a quantity of 250 records, customers are willing to pay \$10 a record (this comes from plugging 250 into the inverse demand function). Since the firm has no costs, its profits are simply equal to its revenues of \$10 times 250 records, or \$2500.

(b) Determine the admission fee the store will charge, the price it will charge per record, and the total profits the store will make if it takes the second approach.

This second approach is a two-part tariff. Once choosing a price for the records, the store can charge an admission fee equal to the amount of consumer surplus the customer is left with when buying records at that price. If the fee were set higher than this, the customer would choose not to shop at the store. If the fee is anything less than this, even one penny less, the customer will definitely shop at the store since paying the fee and buying records will leave him with at least some positive surplus. The profits the store will earn will be equal to the profits on the sales of the records plus the admission fee:

$$\pi(y, fee) = p(y) \cdot y + fee$$

Plugging in the inverse demand function gives us:

$$\pi(y, fee) = (20 - \frac{1}{25}y)y + fee$$

$$\pi(y, fee) = 20y - \frac{1}{25}y^2 + fee$$

Now we want to plug in an expression for the fee. Note that the store will set the fee equal to the consumer surplus at a quantity of y. This is the area under the demand curve above the price $p(y)$ up to the quantity y:

$$CS(y) = \frac{1}{2}(p(0) - p(y)) \cdot (y - 0)$$

$$CS(y) = \frac{1}{2}\left(20 - (20 - \frac{1}{25}y)\right) \cdot y$$

$$CS(y) = \frac{1}{50}y^2$$

Plugging this in for the fee in the profit function gives us:

$$\pi(y) = 20y - \frac{1}{25}y^2 + \frac{1}{50}y^2$$

$$\pi(y) = 20y - \frac{1}{50}y^2$$

Now we can find the optimal quantity by taking the derivative of this profit function with respect to y and setting it equal to zero:

$$\frac{d\pi(y)}{dy} = 20 - \frac{2}{50}y$$

$$0 = 20 - \frac{2}{50}y$$
$$\frac{2}{50}y = 20$$
$$y = 500$$

So the new profit-maximizing quantity is 500 records. Notice that at this quantity, the price per record is actually zero; the store is basically giving away the records. However, the reason the store is willing to price the records at zero is because it is maximizing the money it makes off of the admission fee. Plugging the quantity of 500 back into the formula for the consumer surplus gives us a fee of $\frac{1}{50} \cdot 500^2$ or \$5000. The store's profits are equal to this fee. By using a two-part tariff the store has doubled its profits.

(c) A local government official is concerned that an exclusive event is unfair and is considering banning the practice of charging admission for exclusive sales. Would you support or oppose this ban? Be certain to fully explain your answer with a discussion of the welfare implications of the ban.

The admission fee has actually led to the efficient number of records being sold. So from a total surplus standpoint, the admission fee should be allowed. However, the reason the store was willing to sell the efficient quantity of records was that it retained all of the surplus. With the admission fee, consumers were left with no consumer surplus at all. So from an equity standpoint, you may be opposed to the fee but from an efficiency standpoint, the fee should be allowed.

5. (20 points) A baker uses flour, milk and butter to make biscuits. Each biscuit requires one cup of flour, one cup of milk and two tablespoons of butter. This recipe must be followed exactly, otherwise the biscuits will be inedible.

   (a) Write down a function that gives the number of biscuits (B) that can be made as a function of the cups of flour (F), cups of milk (M) and tablespoons of butter (T) used. Note that the baker can make fractions of biscuits (so one and a half cups of flour, one and a half cups of milk and three tablespoons of butter could be used to make one and a half biscuits).

   We need exactly the right combination of ingredients to bake biscuits. Having extra of any one ingredient does not help us. The total number of biscuits produced will be constrained by whichever ingredient is in short supply. This situation can be modeled with a min function:

$$B = f(F, M, T)$$

$$B = \min(a \cdot F, b \cdot M, c \cdot T)$$

   Now we need to determine what coefficients to have in front of each input. Every biscuit requires one cup of flour, one cup of milk and two tablespoons of butter. So we can never produce more than either the number of cups of flour we have, the number of cups of milk we have, or one half the number of tablespoons of butter we have. So the coefficients on these inputs should be one, one and one half, respectively:

$$B = \min(F, M, \frac{1}{2}T)$$

   (b) Assume that flour, milk and butter are all variable inputs. The price of a cup of flour is \$1, the price of a cup of milk is \$2 and the price of a tablespoon of butter is \$0.50. Derive a function giving the bakers minimum costs as a function of the number of biscuits baked (B). Your function should contain only numerical constants and B, there should be no other variables in your function.

   For each biscuit we make, we need to use one cup of flour, one cup of milk and two tablespoons of butter, no more and no less. So the marginal cost of each biscuit will be equal to the cost of these ingredients:

$$MC(B) = p_F \cdot 1 + p_M \cdot 1 + p_T \cdot 2$$

$$MC(B) = 1 \cdot 1 + 2 \cdot 1 + \frac{1}{2} \cdot 2$$

$$MC(B) = 4$$

   So each biscuit requires \$4 in ingredients. Therefore our total cost of making B biscuits will be $4B$. This gives us our total cost function (assuming there are no fixed costs):

$$C(B) = 4B$$

(c) Construct a graph showing the baker's demand for flour as a function of the price of flour. Your graph should have cups of flour (F) on the horizontal axis and the price of a cup of flour (p_F) on the vertical axis. You should label any kinks, intercepts and slopes with their exact numerical values if possible.

This is a bit tricky. First let's think about how the firm's optimal quantity depends on the price of flour. Keeping the price of milk and butter at \$2 and \$0.50, the marginal cost of a biscuit will be:

$$MC(B) = p_F \cdot 1 + 2 \cdot 1 + \frac{1}{2} \cdot 2$$

$$MC(B) = 3 + p_F$$

So the marginal costs will be constant. If this marginal cost is below the price of a biscuit, the baker will make a profit on each biscuit and therefore want to make as many biscuits as possible. Demand for flour will go to infinity. If the price of a biscuit is lower than this marginal cost, then no biscuit will be profitable and demand for flour will be zero. When the price of a biscuit is exactly equal to the marginal cost of a biscuit, the baker will break even on each biscuit and therefore be indifferent between how much flour is used and how many biscuits are produced. So the demand for flour is zero when $p_B < 3 + p_F$, infinity when $p_B > 3 + p_F$ and can be anything when $p_B = 3 + p_F$. This gives us the demand curve shown below.