Econ 303 - Intermediate Microeconomic Theory
College of William and Mary
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Final Exam - Solutions

You have until 3:30pm to complete the exam, be certain to use your time wisely. Answer all questions directly on the exam. You must show all of your work to receive full credit. Non-graphing calculators may be used (no graphing calculators or phones can be used). You may leave answers as fractions. Unless a problem says otherwise, you can assume that firms can produce fractions of units and charge non-integer prices (so a firm could produce 82.4 units and sell at a price of $5.325 per unit). Remember to put your name on the exam. Good luck!

Name: **ID Number:**

1. (20 points) Busch Gardens is the only local theme park and is deciding how much to charge for each ride on a roller coaster. The marginal cost of letting one additional person ride a roller coaster is zero. The demand for rides is given by the following inverse demand function:

$$p(R) = 5 - \frac{1}{100}R \qquad (1)$$

where R is the total number of rides.

(a) Suppose that Busch Gardens decides to sell tickets for individual rides (rather than charge an admission fee). What is the profit-maximizing price per ride Busch Gardens will charge and how many tickets will be sold?

Busch Gardens will sell up to the point where the marginal cost of an additional ride just equals the marginal revenue of an extra ride. For every ride before this, the marginal revenue will exceed the marginal cost leading to an increase in profits if the ride is sold. For every ride after this, the marginal cost exceeds the marginal revenue so the ride would lead to a decrease in profits if sold. To find the quantity at which marginal revenue equals marginal cost, we first need to derive the marginal revenue function:

$$R(R) = p(R) \cdot R$$

$$R(R) = (5 - \frac{1}{100}R)R$$

$$R(R) = 5R - \frac{1}{100}R^2$$

$$MR(R) = \frac{dR(R)}{dR} = 5 - \frac{1}{50}R$$

Now we can find the monopoly quantity by setting this marginal revenue function equal to marginal costs:

$$MR(R) = MC(R)$$

$$5 - \frac{1}{50}R = 0$$

$$\frac{1}{50}R = 5$$
$$R_m = 250$$

The monopoly price can by found by plugging this quantity into the inverse demand function:

$$p(250) = 5 - \frac{1}{100} \cdot 250$$
$$p(250) = 2.5$$

So Busch Gardens will sell 250 ride tickets at a price of \$2.50 a ticket.

(b) What is the efficient quantity of rides? Calculate the deadweight loss associated with the outcome in part (a).

The efficient quantity of tickets would be the number at which marginal benefit equals marginal cost. Every ticket for which marginal benefit exceeds marginal cost would increase total surplus while every ticket for which marginal cost exceeds marginal benefit would decrease total surplus. So the efficient quantity can be found by setting marginal cost equal to marginal benefit (which is equivalent to the inverse demand curve):

$$MC(R) = p(R)$$
$$0 = 5 - \frac{1}{100}R$$
$$R_{eff} = 500$$

The deadweight loss from being at R_m instead of R_{eff} is equal to the surplus on those tickets between R_m and R_{eff} that do not get sold. This is the triangular area between the marginal cost curve and the demand curve from R_m to R_{eff}:

$$DWL = \frac{1}{2}(p(R_m) - MC(R_m))(R_{eff} - R_m)$$
$$DWL = \frac{1}{2}(2.5 - 0)(500 - 250)$$
$$DWL = 312.5$$

(c) Suppose that Busch Gardens decides to use a two part tariff, charging an admission fee and then selling ride tickets. In order to maximize profits, what should Busch Gardens charge in admission fees and what should it charge for each ride ticket? Note: You can think of the demand curve as the demand curve for a single customer, in which case you are finding the admission fee charged to that one customer and the price per ride ticket for that customer.

Now Busch Gardens can capture all of the total surplus: the standard producer surplus through the ticket sales and the consumer surplus through the fee. Busch Gardens will therefore maximize profits by maximizing total surplus. This is exactly what the efficient quantity does. So Busch Gardens will sell the efficient quantity of tickets, 500, and charge the price people are willing to pay at the efficient quantity, zero. The fee can then be set equal to the consumer surplus at this quantity and price:

$$fee = CS$$
$$fee = \frac{1}{2}(p(0) - 0)(R_{eff} - 0)$$
$$fee = \frac{1}{2}(5 - 0)(500 - 0)$$
$$fee = 1250$$

So Busch Gardens will charge an admission fee of $1250 and then charge nothing for the ride tickets.

2. (20 points) A firm has two different factories that it can use to produce notebook computers, factory A and factory B. The total cost and marginal cost functions for these two factories are given by:

$$C_A(N_A) = 1000 + 10N_A^2 \tag{2}$$

$$MC_A(N_A) = 20N_A \tag{3}$$

$$C_B(N_B) = 2000 + 20N_B^2 \tag{4}$$

$$MC_B(N_B) = 40N_B \tag{5}$$

where N_A is the number of notebooks produced in factory A and N_B is the number of notebooks produced in factory B.

(a) If the firm is going to produce a total of N notebooks, what fraction of these notebooks should it produce at factory A and what fraction should it produce at factory B in order to minimize costs? You can assume that the firm has to pay any fixed costs associated with a factory even if it produces zero notebooks at that factory.

The firm will allocate production across the two factories such that the marginal costs are equal. If the marginal costs were higher at one factory than the other, the firm could save money by shifting output from the high marginal cost factory to the low marginal cost factory. Therefore, the firm will choose N_A and N_B such that:

$$MC_A(N_A) = MC_B(N_B)$$

$$20N_A = 40N_B$$

$$N_A = 2N_B$$

So the firm will produce twice as much at factory A as at factory B. To find these outputs in terms of total output N, we can use the fact that the sum of the outputs must equal N:

$$N = N_A + N_B$$

Plugging in our result from above gives us:

$$N = 2N_B + N_B$$

$$N_B = \frac{1}{3}N$$

$$N_A = 2N_B = \frac{2}{3}N$$

So the firm will produce two thirds of its output at factory A and one third at factory B.

(b) Derive an expression $C(N)$ giving the total costs of producing N notebooks assuming the firm divides output between the factories in the optimal way.

The firm's total costs will be the sum of the costs at factory A and factory B:

$$C(N_A, N_B) = C_A(N_A) + C_B(N_B)$$

Plugging in the results from part (a) will let us rewrite this solely in terms of N:

$$C(N) = C_A(\frac{2}{3}N) + C_B(\frac{1}{3}N)$$

$$C(N) = 1000 + 10\left(\frac{2}{3}N\right)^2 + 2000 + 20\left(\frac{1}{3}N\right)^2$$

$$C(N) = 3000 + \frac{20}{3}N^2$$

(c) Find the firm's supply function, $N(p)$, assuming that the industry for notebook computers is perfectly competitive.

If the firm is in a competitive industry, its supply curve will be the same as its marginal cost curve above the shutdown price. Let's first find the shutdown price by setting marginal cost equal to average variable cost (this finds the minimum of the average variable cost curve, if price were to drop below this point the revenue on each unit wouldn't even cover the average variable costs let alone the average fixed costs):

$$MC(N) = AVC(N)$$

Given our cost function from part (b), this becomes:

$$\frac{40}{3}N = \frac{20}{3}N$$

The only N that solves this is N equal to zero. Plugging N equal to zero back into either $MC(N)$ or $AVC(N)$ will give us the shutdown price:

$$p_{SD} = MC(0) = \frac{40}{3} \cdot 0$$

$$p_{SD} = 0$$

So the firm will produce at all positive prices and will produce at the point where price equals marginal cost:

$$p = MC(N)$$

$$p = \frac{40}{3}N$$

Rearranging this equation gives us the supply function for the firm:

$$N(p) = \frac{3}{40}p$$

(d) How would your answer to parts (a) and (b) change if instead of the total cost and marginal cost functions above you were told that the factory total cost functions were $C_A(N_A) = 1000 + 10N_A$ and $C_B(N_B) = 2000 + 20N_B$? Be as specific as possible.

The big difference now is that the marginal costs at each factory are constant and lower for factory A than factory B: $MC_A(N_A) = 10$ and $MC_B(N_B) = 20$. So no matter how output is currently divided, it will always be cheaper to produce the next unit at factory A. Consequently, the firm will produce all output at factory A leading to the following new answers for parts (a) and (b):

$$N_A = N$$

$$N_B = 0$$

$$C(N) = C_A(N) + C_B(0)$$

$$C(N) = 1000 + 10N + 2000 + 20 \cdot 0$$

$$C(N) = 3000 + 10N$$

3. (20 points) Each firm in a competive industry has the following total cost and marginal cost functions:

$$C_i(y_i) = 160 + 10y_i^2 \tag{6}$$

$$MC_i(y_i) = 20y_i \tag{7}$$

where y_i is that individual firm's output. Demand for the industry is given by the following inverse demand function:

$$p(y) = 100 - \frac{2}{5}y \tag{8}$$

(a) Find the shutdown price and the breakeven price for an individual firm.

A firm will shutdown if the price is below the minimum of the average variable cost curve. At these prices, the firm's revenue would not cover all of its variable costs or any of its fixed costs so it would be better off simply shutting down and just losing its fixed costs. To find the minimum of the average variable cost curve, we can find where it intersects the marginal cost curve (knowing that the two always intersect at the minimum of the average variable cost curve):

$$MC(y) = AVC(y)$$
$$20y = 10y$$
$$y = 0$$

Plugging this quantity back into either $MC(y)$ or $AVC(y)$ will give us the shutdown price:

$$p_{SD} = MC(0)$$
$$p_{SD} = 0$$

The firm will break even when the price hits the minimum of the average total cost curve. We can find this point using the same approach we took for the shutdown price:

$$MC(y) = AC(y)$$
$$20y = \frac{160}{y} + 10y$$
$$10y = \frac{160}{y}$$
$$y^2 = 16$$
$$y = 4$$
$$p_{BE} = MC(4)$$
$$p_{BE} = 20 \cdot 4$$
$$p_{BE} = 80$$

So an individual firm will produce at all positive prices but will only earn positive profits for prices above \$80.

(b) Suppose there are currently 50 firms in the industry. What will the each firm's profits be in the short run?

To find firm profits, we need to know what the equilibrium price will be. To get this, we need to find the industry supply curve. An individual firm will supply where price is equal to marginal cost:

$$p = MC(y_i)$$

$$p = 20y_i$$

Rearranging this gives us the individual firm's supply function:

$$y_i(p) = \frac{1}{20}p$$

Given that there are 50 firms, industry supply will be:

$$S(p) = 50 \cdot y_i(p)$$

$$S(p) = \frac{5}{2}p$$

It will be useful to rearrange this industry supply function to get the inverse supply function:

$$p_S(y) = \frac{2}{5}y$$

Now we can find the equilibrium quantity by setting this inverse supply function equal to the inverse demand function:

$$p_S(y) = p_D(y)$$

$$\frac{2}{5}y = 100 - \frac{2}{5}y$$

$$\frac{4}{5}y = 100$$

$$y = 125$$

Plugging this quantity back into the inverse demand function will give us the equilibrium price:

$$p(125) = 100 - \frac{2}{5} \cdot 125$$

$$p = 50$$

Individual firm profits will therefore be:

$$\pi_i = p \cdot y_i - C_i(y_i)$$

$$\pi_i = 50 \cdot (\frac{1}{20} \cdot 50) - (160 + 10(\frac{1}{20} \cdot 50)^2)$$

$$\pi_i = -97.5$$

(c) What will the number of firms be in the long run? Note: You can have fractions of firms.

Given that firms are losing money, firms will exit the industry driving up price until it reaches the breakeven price. At this point, the industry will be in long run equilibrium. At the breakeven price, demand is:

$$80 = 100 - \frac{2}{5}y$$

$$\frac{2}{5}y = 20$$

$$y = 50$$

We already found that an individual firm's supply at the breakeven price is 4 units. So the total number of firms must be:

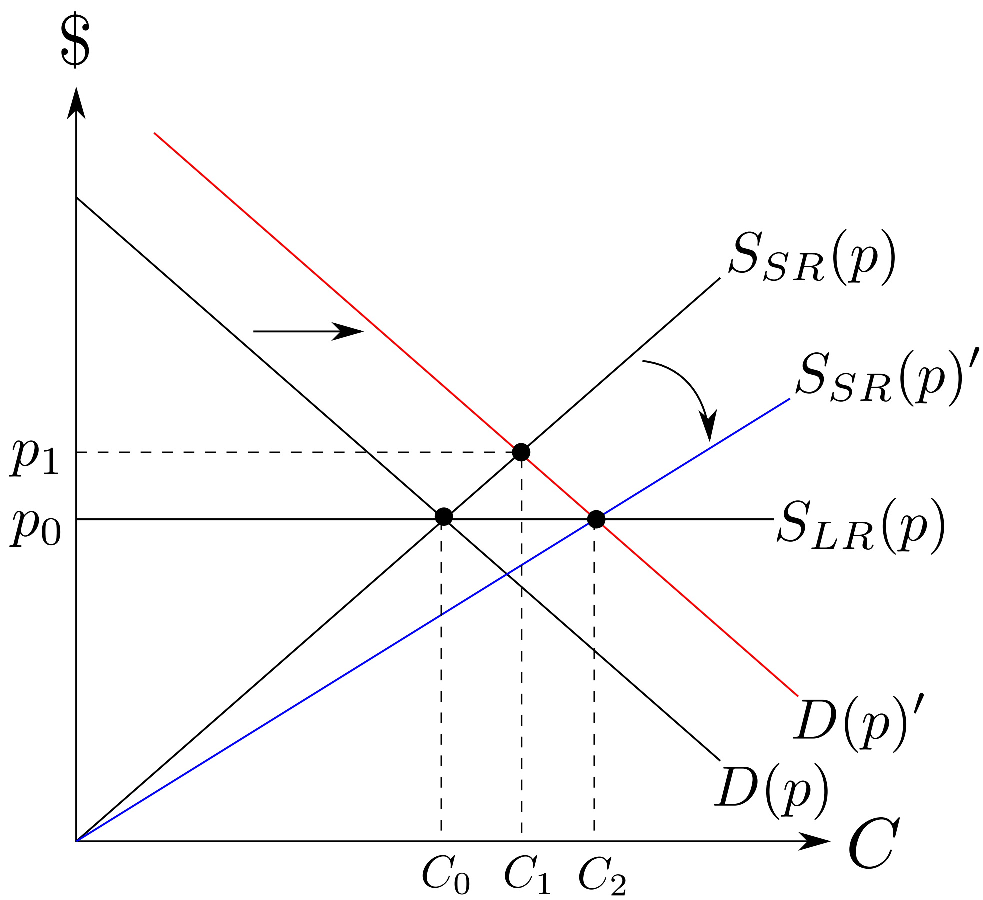
$$n = \frac{y_{total}}{y_i}$$

$$n = \frac{50}{4}$$

$$n = \frac{25}{2}$$

There will be 12.5 firms in the long run.

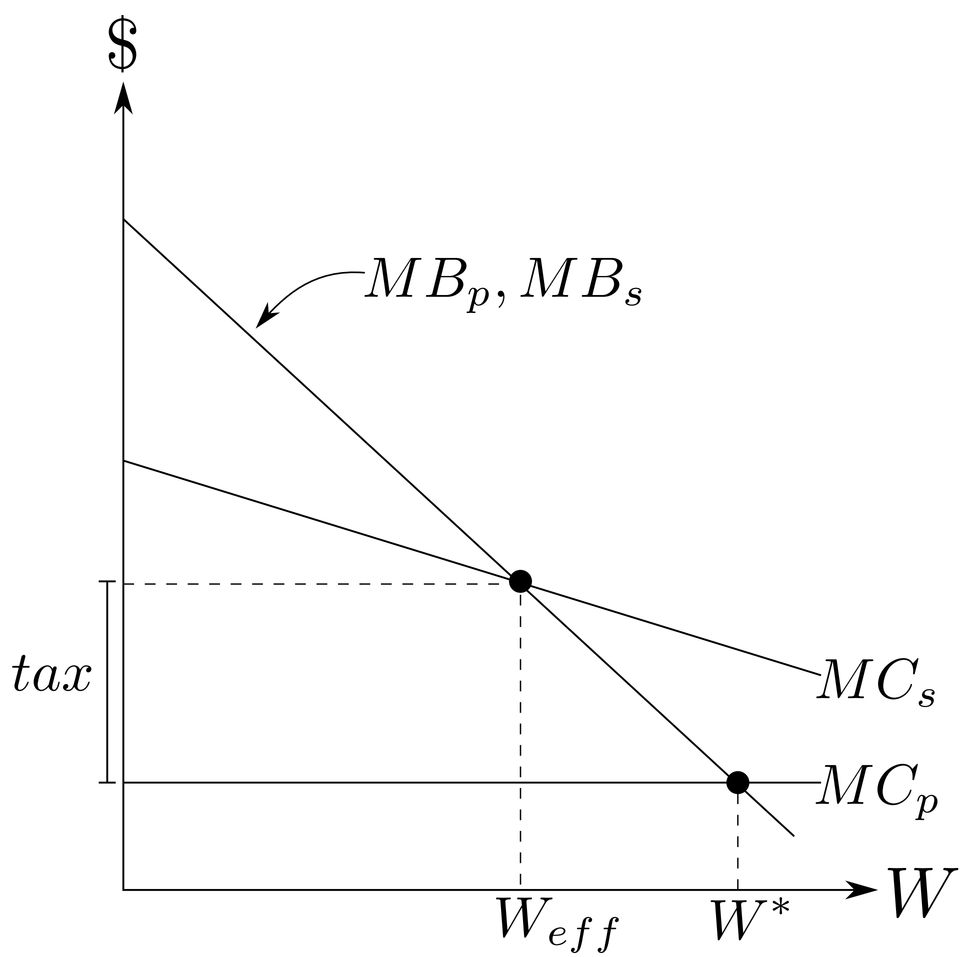
4. (15 points) The graph below shows the supply and demand curves for the market for coffee. Note that both the short run and long run supply curves are given. The market for coffee is highly competitive. The market is currently at the long run equilibrium price and quantity. Suppose that a news report is released showing that coffee has greater nutritional benefits than previously thought. Show the short run and long run effects of this news report on the price and quantity of coffee sold. Be certain to clearly label any relevant points and curves you add to the graph.



Before the news report, the market is at the initial long run equilibrium price and quantity p_0 and C_0. The news report will change consumer preferences, leading to higher demand for coffee at any given price. This shifts the demand curve to the right to $D(p)'$. Initially, this causes a movement along the short run supply curve to the new short run equilibrium price and quantity p_1 and C_1. At this new higher price firms will be making positive profits. These positive profits will induce firms to enter the industry, rotating the short run supply curve out until it intersects the new demand curve at the breakeven price. At this new number of firms, the short run supply curve is $S_{SR}(p)'$, bringing us to the new long run eqilibrium price and quantity of p_0 (same as the initial price) and C_2.

5. (15 points) In the market for speakers, customers have a linear, downward sloping demand curve for watts (a rough measure of how powerful the speakers are). Speaker companies have total costs that increase linearly with watts leading to constant marginal costs. Speakers generate a negative externality for neighbors. Each extra watt causes harm to neighbors from the extra noise. However, the marginal impact of this harm is gets smaller as the number of watts increase. Draw a graph below that has watts (W) on the horizontal axis and dollars on the vertical axis showing the following:

   - The private and social marginal benefit curves for watts.
   - The private and social marginal cost curves for watts.
   - The equilibrium number of watts in a competitive market with no government intervention.
   - The efficient number of watts.
   - The size of a quantity tax on watts that would lead to the efficient number of watts.



Given that there are no positive externalities in this case, the private marginal benefit and social marginal benefit curves are actually the same curve which should be a downward sloping line (labelled MB_p, MB_s on the graph). The private marginal cost curve should be a horizontal line (MC_p on the graph). The social marginal cost curve should lie entirely above the private marginal cost curve (since it includes the private marginal costs plus the externality) and should slope downward because the negative externality portion of the social marginal costs is decreasing as W gets larger. The equilibrium quantity in the absence of any government intervention will be where the private marginal benefit curve intersects the private marginal cost curve. This occurs at the quantity W^* on the graph. The socially efficient quantity will be where the social marginal benefit and social marginal cost curves intersect, W_{eff} on the graph. The quantity tax that would get us to this quantity is equal to the gap between the social marginal costs and the the private marginal costs at this socially efficient quantity.

6. (10 points) There are two farmers selling oranges at the farmers market. The two farmers each decide how many oranges to bring and then the price that they can sell their oranges for is determined by the market demand curve:

$$D(p) = 100 - 25p \tag{9}$$

where $D(p)$ is the total number of oranges bought from both farmers combined at the price p. Farmer A's costs go up by \$2 for each extra orange he brings to the farmers market. Farmer B's costs go up by \$1 for every extra orange she brings to the farmers market. Find an expression giving farmer A's optimal number of oranges to bring to the market (O_A) as a function of the number of oranges farmer B brings to the market (O_B).

Farmer A will choose the quantity that maximizes profits which are a function of both O_A and O_B:

$$\pi_A(O_A, O_B) = p(O_A + O_B) \cdot O_A - C_A(O_A)$$

Using the demand equation given in the problem, we can rewrite this as:

$$\pi_A(O_A, O_B) = (4 - \frac{1}{25}(O_A + O_B)) \cdot O_A - C_A(O_A)$$

From the problem, we know that each extra orange increases farmer A's costs by \$2, implying that the total cost function is $C_A(O_A) = 2O_S$. (Note that we don't know if there are any fixed costs but we don't really care, fixed costs will not make a difference when thinking about the profit of bringing an additional orange. This marginal profit will only depend on price and marginal cost.) This allows us to rewrite the profit function as:

$$\pi_A(O_A, O_B) = (4 - \frac{1}{25}(O_A + O_B)) \cdot O_A - 2O_A$$

$$\pi_A(O_A, O_B) = 2O_A - \frac{1}{25}O_A^2 - \frac{1}{25}O_AO_B$$

This function will be maximized where its derivative is equal to zero (note that we are taking the derivative with respect to O_A as that is the variable Farmer A gets to choose):

$$0 = \frac{d\pi}{dO_A}$$

$$0 = 2 - \frac{2}{25}O_A - \frac{1}{25}O_B$$

$$O_A = 25 - \frac{1}{2}O_B$$

This is the best response function for Farmer A; it gives the profit maximizing O_A for any particular value of O_B.