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## Midterm 2 - Solutions

You have until 3pm to complete the exam, be certain to use your time wisely. For multiple choice questions, mark your answer on your scantron sheet. Choose only one answer for each multiple choice question; if more than one letter is filled in for a question it will be marked wrong. For the short answer questions, write your answers directly on the exam. Show your work clearly, place a box around final answers and be certain to label any graphs you draw. Final answers may be left as fractions. Non-graphing calculators may be used but they should not be necessary. Good luck!

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**Name:**

**ID Number:**

**Section:**

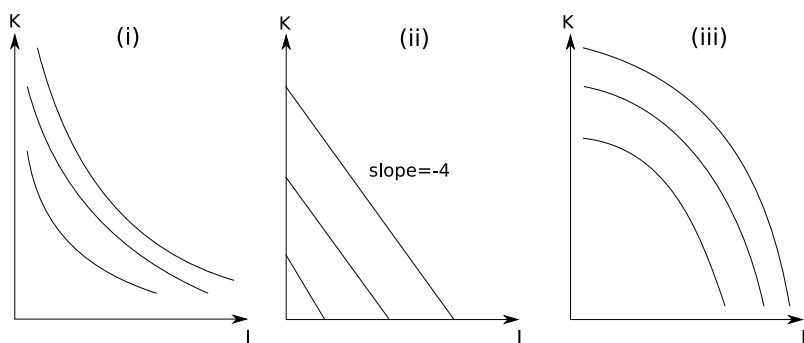
### SECTION I: MULTIPLE CHOICE (60 points)

1. Suppose that a firm's production function exhibits increasing returns to scale. If it costs the firm \$100 to produce 20 units of output, then it will cost the firm:
  - (a) Less than \$100 to produce 40 units of output.
  - (b) Exactly \$200 to produce 40 units of output.
  - (c) More than \$200 to produce 40 units of output.
  - (d) Less than \$200 to produce 40 units of output.

(d) To double output from 20 to 40 units, inputs will increase, increasing costs, but they will less than double. So the costs of producing 40 units will be greater than the costs of producing 20 units but less than double the costs of producing 20 units.
2. Suppose that apples are a Giffen good. If the price of apples increases, then:
  - (a) The income and substitution effects for apples will both be positive.
  - (b) The income and substitution effects for apples will both be negative.
  - (c) The income effect will be positive and the substitution effect will be negative for apples.
  - (d) The income effect will be negative and the substitution effect will be positive for apples.

(c) The substitution effect will be negative when the price of a good increases, even if it is a Giffen good. Since apples are a Giffen good, we know that the overall change in apples will be positive when the price increases. Given that the substitution effect is negative, this means that the income effect must be positive.
3. A firm that uses capital and labor is operating at its profit-maximizing input levels given the current prices of inputs and output. Which of the following price changes would not lead to an increase in labor hired in the short run? (Assume capital is fixed in the short run.)
  - (a) An increase in the price of output and the price of capital.
  - (b) An increase in the price of output and a decrease in the price of capital.
  - (c) An increase in the price of labor and a decrease in the price of capital.
  - (d) A decrease in the price of labor and an increase in the price of capital.

- (c) The price of the fixed good (capital) will not effect short run demand for labor. What will influence the demand for labor are changes in the price of labor or the price of output. If the price of output goes up, demand for labor will increase. If the price of labor goes up, demand for labor will decrease.
4. At the current price, the price elasticity of demand for comic books is equal to  $-2$ . If the only comic book store in town raises the price of comic books by a small amount, we would expect:
- The number of comic books sold to increase and total revenues of the store to increase.
  - The number of comic books sold to decrease and total revenues of the store to increase.
  - The number of comic books sold to increase and total revenues of the store to decrease.
  - The number of comic books sold to decrease and total revenues of the store to decrease.
- (d) If the price elasticity of demand is negative, an increase in price will lead to a decrease in demand. So the number of comic books sold will decrease. Because demand is currently elastic, an increase in price will lead to a decrease in revenue for the comic book store. To increase revenues, the store should be lowering the price of comic books until demand is unit elastic.



Refer to the figures above to answer questions 5 and 6. Each of the above graphs shows three isoquants for a particular production technology. You can assume that all three technologies are monotonic (increasing an input will increase output).

5. Which of the graphs could correspond to the isoquants for the production function:  $f(K, L) = K^3L^2$ ?
- (i).
  - (ii).
  - (iii).
  - (i) and (iii).

(a) Notice that the technical rate of substitution for the production function is:

$$TRS = -\frac{MPL}{MPK} = -\frac{2K^3L}{3K^2L^2} = -\frac{2K}{3L}$$

This technical rate of substitution gets smaller in magnitude as capital decreases and labor increases. The only graph with isoquants displaying a diminishing technical rate of substitution is the first one.

6. Suppose that you are told that the wage is 5 times as large as the rental rate of capital. Which technologies could lead a firm to use positive amounts of both capital and labor in the long run under these prices?

- (a) (i).
- (b) (ii).
- (c) (i) and (iii).
- (d) (i) and (ii).

(a) The information in the question reveals that the isocost lines have a slope of -5. An isocost curve of slope -5 could certainly be tangent to an isoquant on the first graph in the interior. On the second graph, an isocost curve will never be tangent to an isoquant. The isoquants will always be flatter leading a firm to use only capital and no labor (a unit of labor is four times as productive as a unit of capital but costs five times as much). For the third graph, there will be a point of tangency between an isocost curve and an isoquant in the interior, but this point will not be optimal. A firm could reach a lower isocost curve by moving away from the point of tangency to one of the endpoints of the isoquant.

7. The deadweight loss generated by a quantity tax will be zero if:

- (a) Supply is perfectly elastic and demand is downward sloping.
- (b) Supply is perfectly inelastic and demand is downward sloping.
- (c) (a) and (b).
- (d) Neither (a) nor (b).

(b) If supply is perfectly inelastic (the supply curve is a vertical line) the quantity sold is the same before and after the tax. Producer's revenues decrease by the amount of the tax times the quantity of output which is exactly equal to tax revenues. Consumer surplus does not change. So there is no change in total surplus and no deadweight loss generated. For a perfectly elastic supply (a horizontal supply curve), sellers will receive the same price as before, consumers' price will increase by the size of the tax, and the quantity sold will decrease. This decrease in the quantity sold will generate a deadweight loss equal to what the consumer surplus was on those units that are no longer being sold.

8. A firm with standard, convex isoquants is currently producing 50 cars using 10 machines and 10 workers. At this combination of inputs, the marginal product of labor is 5 and the marginal product of capital is 10. The wage is \$10 and the rental rate of capital is \$5. The firm can lower total costs and still produce 50 cars by:

- (a) Increasing the amount of labor and decreasing the amount of capital used.
- (b) Increasing the amount of capital and decreasing the amount of labor used.
- (c) Increasing the amount of capital and increasing the amount of labor used.
- (d) The firm is already minimizing costs.

(b) Currently, the marginal product of labor relative to the marginal product of capital is less than the price of labor relative to the price of capital. Therefore, if the firm moves along their isoquant by substituting capital for labor, the money saved on labor will be greater than the additional money spent on capital.

9. Suppose that books are an inferior, ordinary good. When the price of books goes up, the magnitude of the income effect for books will be \_\_\_\_\_ the magnitude of the substitution effect for books.
- (a) Greater than.
  - (b) Less than.
  - (c) Equal to.
  - (d) Not enough information.
- (b) When the price of books increases, the overall number of books purchased should decrease because they are ordinary. We know that the substitution effect will be negative for books when the price of goods goes up. Because books are inferior, the income effect will be positive. To get a net decrease in the demand for books the substitution effect must be larger in magnitude than the income effect.
10. When the demand curve is a downward sloping line and the supply curve is an upward sloping line, which of the following will be true as the magnitude of a quantity tax increases?
- (a) Tax revenues will first increase and then decrease, deadweight loss will increase.
  - (b) Deadweight loss will first increase and then decrease, tax revenues will increase.
  - (c) Both tax revenues and deadweight loss will first increase and then decrease.
  - (d) Both tax revenues and deadweight loss will increase.
- (a) As the size of the tax increases, initially the increase in tax revenue from the additional tax per unit will more than offset the decrease in tax revenue from fewer units sold and overall tax revenues will increase. Eventually, the increased revenue from higher tax per unit will be less than the decreased revenue from fewer units sold and tax revenues will begin to decrease. Deadweight loss will be strictly increasing as the tax increases and more consumer and producer surplus lost from fewer and fewer units being sold.
11. Suppose that in the short run, capital is a fixed input, labor is a variable input and electricity is a quasi-fixed input. Which of the following is not true?
- (a) Changes in the price of labor may affect the optimal level of labor hired in the short run.
  - (b) Changes in the price of capital may affect the optimal level of labor hired in the short run.
  - (c) Changes in the price of electricity may affect the decision to shut down in the short run.
  - (d) Firm profits will depend on the price of capital in the short run.
- (b) The price of a fixed good will influence the level of profits but will not affect the choice of a variable input. The price of a quasi-fixed good may affect the choice of a variable input because it is relevant to the firm's decision about whether or not to operate in the short run.
12. Suppose there are three consumers in the market for ice cream and all three have identical, downward sloping linear demand curves with a slope of  $-3$  (note that this is the slope of the demand curve on a graph with price on the vertical axis and quantity on the horizontal axis). The magnitude of the slope of the market demand curve will be:
- (a) Less than 3.

- (b) Equal to 3.
- (c) Equal to 9.
- (d) Greater than 9.

(a) The market demand curve will be flatter than the individual demand curves. In this case, notice that when the price of ice cream drops by \$3, each consumer buys one more unit of ice cream (this is what the slope of  $-3$  means). So market demand will go up by three units of ice cream when the price drops by \$3. This implies a slope of  $-1$  for the market demand curve.

13. Suppose a firm's production technology is given by  $f(K, L) = 10K^2 + 5L^{\frac{1}{2}}$ . Which of the following statements is true?
- (a) The technology exhibits diminishing marginal product of capital and diminishing marginal product of labor.
  - (b) The technology exhibits increasing marginal product of capital and diminishing marginal product of labor.
  - (c) The technology exhibits diminishing marginal product of capital and increasing marginal product of labor.
  - (d) The technology exhibits increasing marginal product of capital and increasing marginal product of labor.

(b) The marginal product of labor and the marginal product of capital are given by the following equations:

$$MPL = \frac{5}{2}L^{-\frac{1}{2}}$$

$$MPK = 20K$$

Notice that the marginal product of labor gets smaller as  $L$  increases and the marginal product of capital gets larger as  $K$  increases.

14. What must be true about a firm's short run and long run profits (assuming that the firm is maximizing profits and that all prices of inputs and outputs are fixed)?
- (a) Profits will be the same in the short run and long run.
  - (b) Profits will always be larger in the long run than in the short run.
  - (c) Profits will always be larger in the short run than in the long run.
  - (d) Long run profits will be greater than or equal to short run profits.

(d) We know that profits should be at least as large in the long run as they are in the short run because anything the firm could do in the short run, it can also do in the long run. Short run and long run profits will be equal if the fixed inputs in the short run happened to be fixed at their optimal long run levels.

15. Suppose that output as a function of capital (holding other inputs fixed) is graphed with output on the vertical axis and capital on the horizontal axis. The slope of this curve will be equal to:
- (a) The marginal product of capital.
  - (b) The marginal product of labor.
  - (c) The technical rate of substitution.

- (d) The wage divided by the rental rate of capital.
- (a) The slope of this curve is the change in output relative to a change in capital. This is the definition of the marginal product of capital.

## SECTION II: SHORT ANSWER (40 points)

For this section, be certain to show your work and clearly label any graphs you draw. Give complete answers but keep them concise. Please place a box around final answers where appropriate.

1. Brendan consumes only apples ( $A$ ) and bananas ( $B$ ). His demand functions for apples and bananas in terms of the income ( $I$ ), the price of apples ( $p_A$ ) and the price of bananas ( $p_B$ ) are given below:

$$A(I, p_A, p_B) = \frac{I}{p_A + \frac{p_A^2}{p_B}} \quad (1)$$

$$B(I, p_A, p_B) = \frac{I}{p_B + \frac{p_B^2}{p_A}} \quad (2)$$

- (a) Suppose that Brendan's income is \$120, the price of apples is \$1 and the price of bananas is \$1. How many apples and bananas does Brendan consume? (3 points)

We simply need to plug the prices and income into the demand functions to get Brendan's consumption bundle:

$$A(120, 1, 1) = \frac{120}{1 + \frac{1^2}{1}} = 60$$

$$B(120, 1, 1) = \frac{120}{1 + \frac{1^2}{1}} = 60$$

- (b) Suppose that the price of apples increases to \$2. How many apples and bananas does Brendan consume after the price change? (3 points)

Using the new price of apples in our demand functions gives us:

$$A(120, 2, 1) = \frac{120}{2 + \frac{2^2}{1}} = 20$$

$$B(120, 2, 1) = \frac{120}{1 + \frac{1^2}{2}} = 80$$

- (c) Calculate the change in Brendan's demand for bananas due to the income effect and the change in Brendan's demand for bananas due to the substitution effect. (10 points)

Before we can calculate the income and substitution effects, we need to find the intermediate budget line that would make our old bundle just affordable under the new prices. This amounts to finding the income required to buy 60 apples and 60 bananas at the new prices:

$$I' = 2 \cdot 60 + 1 \cdot 60 = 180$$

Now we can find our intermediate bundle using this new income and the new prices:

$$A(180, 2, 1) = \frac{180}{2 + \frac{2^2}{1}} = 30$$

$$B(180, 2, 1) = \frac{180}{1 + \frac{1}{2}} = 120$$

Now we can get the income and substitution effects for bananas by comparing the different bundle:

$$\Delta B_{sub} = B_{int} - B_{old} = 120 - 60 = 60$$

$$\Delta B_{inc} = B_{new} - B_{int} = 80 - 120 = -40$$



2. A firm uses nuts ( $N$ ) and bolts ( $B$ ) to produce widgets ( $w$ ). The production function for the firm is the following:

$$f(N, B) = 10N^{\frac{1}{2}}B^{\frac{1}{2}} \quad (3)$$

- (a) Derive expressions for the marginal product of nuts, the marginal product of bolts, and the technical rate of substitution. (4 points)

$$MP_N = \frac{df(N, B)}{dN} = \frac{1}{2}10N^{-\frac{1}{2}}B^{\frac{1}{2}} = 5N^{-\frac{1}{2}}B^{\frac{1}{2}}$$

$$MP_B = \frac{df(N, B)}{dB} = \frac{1}{2}10N^{\frac{1}{2}}B^{-\frac{1}{2}} = 5N^{\frac{1}{2}}B^{-\frac{1}{2}}$$

$$TRS = -\frac{MP_N}{MP_B} = -\frac{5N^{-\frac{1}{2}}B^{\frac{1}{2}}}{5N^{\frac{1}{2}}B^{-\frac{1}{2}}} = -\frac{B}{N}$$

- (b) In the short run, the number of nuts is fixed at 100 but the firm can choose any number of bolts to use. If the price of nuts is \$20, the price of bolts is \$5, and the price of output is \$1, how many bolts does the firm use and how many widgets does it produce in the short run? (10 points)

In the short run, the firm will choose the number of bolts such that the value of the marginal product of a bolt is equal to the cost of a bolt:

$$p_y MP_B = p_B$$

$$p_y \cdot 5N^{\frac{1}{2}}B^{-\frac{1}{2}} = p_B$$

$$1 \cdot 5 \cdot 100^{\frac{1}{2}}B^{-\frac{1}{2}} = 5$$

$$10 = B^{\frac{1}{2}}$$

$$B = 100$$

$$f(100, 100) = 10 \cdot 100^{\frac{1}{2}} \cdot 100^{\frac{1}{2}} = 1000$$

So the firm will use 100 bolts in the short run and will produce 1000 widgets.

- (c) In the long run, the firm decides to produce 100 widgets. Prices are all the same as in part (b). What combination of nuts and bolts will the firm use to minimize costs? (10 points)

To determine the cost minimizing combination of nuts and bolts, our first step is to set the slope of the isocost lines equal to the slope of the isoquant:

$$-\frac{p_N}{p_B} = TRS$$

$$-\frac{p_N}{p_B} = -\frac{B}{N}$$

$$N = \frac{p_B}{p_N}B$$

Now we have an equation relating the optimal number of nuts to the optimal number of bolts. We can substitute this into the production function to solve for one of the two inputs in terms of  $y$ :

$$y = 10N^{\frac{1}{2}}B^{\frac{1}{2}}$$

$$y = 10 \left( \frac{p_B}{p_N} B \right)^{\frac{1}{2}} B^{\frac{1}{2}}$$

$$y = 10 \left( \frac{p_B}{p_N} \right)^{\frac{1}{2}} B$$

$$B = \frac{1}{10} \left( \frac{p_N}{p_B} \right)^{\frac{1}{2}} y$$

Plugging this back into our equation for  $N$  in terms of  $B$  gives us the demand for nuts:

$$N = \frac{p_B}{p_N} \left( \frac{1}{10} \left( \frac{p_N}{p_B} \right)^{\frac{1}{2}} y \right)$$

$$N = \frac{1}{10} \left( \frac{p_B}{p_N} \right)^{\frac{1}{2}} y$$

Now that we have equations for the demand for nuts in terms of prices and output and the demand for bolts in terms of prices and output, we simply need to plug in the given prices and output to find the number of nuts and bolts used:

$$B = \frac{1}{10} \left( \frac{20}{5} \right)^{\frac{1}{2}} 100 = 20$$

$$N = \frac{1}{10} \left( \frac{5}{20} \right)^{\frac{1}{2}} 100 = 5$$

So the firm will use 20 bolts and 5 nuts.