# Midterm 1 - Solutions

You have until 3pm to complete the exam, be certain to use your time wisely. For multiple choice questions, mark your answer on your scantron sheet. Choose only one answer for each multiple choice question; if more than one letter is filled in for a question it will be marked wrong. For the short answer questions, write your answers directly on the exam. Show your work clearly, place a box around final answers and be certain to label any graphs you draw. Final answers may be left as fractions. Non-graphing calculators may be used but they should not be necessary. Good luck!

## Name:

## ID Number:

Section:

# SECTION I: MULTIPLE CHOICE (60 points)

- 1. Suppose that we are told that Alice strictly prefers A to B and B to C. We can then say that Alice prefers A to C if which of the following holds for her preferences?
  - (a) Completeness.
  - (b) Monotonicity.
  - (c) Transitivity.
  - (d) Convexity.

(c) This is basically the definition of transitivity. Transitivity holds if for any bundles A, B, and  $C A \succeq B$  and  $B \succeq C$  implies  $A \succeq C$ .

- 2. Bobby consumes only CDs and DVDs and is currently spending all of his income and buying positive amounts of both goods. At his current consumption bundle, his indifference curve is flatter than his budget line (assume CDs are on the horizontal axis, DVDs are on the vertical axis and that Bobby has standard, convex indifference curves). Bobby can increase his utility by:
  - (a) Buying more CDs and fewer DVDs.
  - (b) Buying more DVDs and fewer CDs.
  - (c) Buying fewer CDs and fewer DVDs.
  - (d) Bobby is already maximizing his utility.

(b) If the indifference curve is convex and flatter than the budget line at the current bundle, there is a portion of the budget line up and to the left of the current bundle that lies above the current indifference curve. So Bobby can move to a higher indifference curve by buying fewer CDs and more DVDs.

- 3. Adam's indifference curves for apples and bananas are upward sloping (you can assume apples are on the horizontal axis and bananas are on the vertical axis). We can say for certain that:
  - (a) Both apples and bananas are good.
  - (b) Both apples and bananas are bad.
  - (c) One of the fruits is a good while the other is a bad.

(d) None of the above.

(c) If the indifference curves are upward sloping, than the change in utility from increasing one good is offset by the change in utility from increasing the other good. This implies that the marginal utilities of the two goods must have opposite signs, making one a good and one a bad. We cannot say which one is good and which one is bad without knowing more information.

- 4. If movie tickets are a 'good' (as opposed to a 'bad'), which of the following statements must be false?
  - (a) Utility graphed as a function of movie tickets has a positive slope.
  - (b) Marginal utility graphed as a function of movie tickets has a positive slope.
  - (c) Utility graphed as a function of movie tickets has a negative slope.
  - (d) Marginal utility graphed as a function of movie tickets has a negative slope.

(c) If movie tickets are good, utility should increase as the number of tickets increases. This means that utility graphed as a function of movie tickets must have a positive slope. We cannot determine the sign of slope for a graph of marginal utility without more information about whether movie tickets exhibit increasing or decreasing marginal utility.

- 5. Assume that peanut butter and jelly are complements (but not perfect complements) and both are normal, ordinary goods. Suppose we have a graph with peanut butter on the horizontal axis and jelly on the vertical axis. The price offer curve we obtain by varying the price of peanut butter will:
  - (a) Have a positive slope.
  - (b) Have a negative slope.
  - (c) Be a vertical line.
  - (d) Be a horizontal line.

(a) As the price of peanut butter goes down, rotating the budget curve away from the axis, the quantity of peanut butter will rise (because peanut butter is an ordinary good) and the quantity of jelly will rise (because jelly and peanut butter are complements).

6. Which of the following utility functions would represent the same preferences?

 $U_A(x,y) = x + y$   $U_B(x,y) = 2x + 2y$   $U_C(x,y) = -2x + -2y$ 

- (a)  $U_A$  and  $U_B$ .
- (b)  $U_A$  and  $U_C$ .
- (c)  $U_B$  and  $U_C$ .
- (d)  $U_A$ ,  $U_B$  and  $U_C$ .

(a) The only difference between  $U_A$  and  $U_B$  is a factor of two. This is a monotonic transformation and would not change the ordering of bundles. The third utility function is also multiplied by a constant but the constant is negative, switching the signs of the marginal utilities leading to very different preferences (the ordering of all bundles is completely reversed).

- 7. Jack's marginal utility from bikes  $(MU_B)$  and his marginal utility from cars  $(MU_C)$  are both constant. If  $MU_B = 4$  and  $MU_C = -2$ , then Jack will (you can assume prices are positive):
  - (a) Buy only bikes.
  - (b) Buy only cars.
  - (c) Buy either bikes or cars depending on the ratio of the prices.
  - (d) Be indifferent between all points on budget line when the  $\frac{p_B}{p_C} = 2$ .

(a) Typically, when both goods have constant marginal utilities we have to compare the MRS to the ratio of the prices to see whether the consumer buys all of one good or all of the other. In this case, one of the goods is good while the other is bad. So no matter what the prices are, buying more cars would always make Jack worse off. So he will spend all of his money on bikes no matter how expensive they are relative to cars.

- 8. Which of the following will definitely not change a person's budget constraint?
  - (a) A doubling of all prices.
  - (b) A doubling of all prices and income.
  - (c) Increasing all prices and income by one dollar.
  - (d) (b) and (c).

(b) Doubling all prices without changing income will shift the budget constraint in. Increasing prices by a dollar each will change the slope of the budget line except in very specific cases (when the prices were originally equal). Doubling both prices and income will have no effect on the budget line (think about what happens with the budget equation, doubling both sides of the equation still leaves you with the same result once you simplify it).

- 9. Suppose that coffee is an inferior good and espresso is a normal good. On a graph with coffee on the horizontal axis and espresso on the vertical axis, the income offer curve will:
  - (a) Have a positive slope.
  - (b) Have a negative slope.
  - (c) Be a horizontal line.
  - (d) Be a vertical line.

(b) As income increases, the quantity of coffee will decrease and the quantity of espresso will increase. So each successive point on the income offer curve will above and to the right of the previous point as income is increased.

#### 10. Dimishing marginal rate of substitution for goods x and y implies:

- (a) Increasing marginal utility for good x.
- (b) Increasing marginal utility for good y.
- (c) Indifference curves that get flatter as you move from left to right.
- (d) All of the above.

(c) The magnitude of the marginal rate of substitution is equal to the magnitude of the slope of the indifference curve. As you move from low values of x to larger values, diminishing MRS implies that this magnitude is getting smaller implying a

flatter and flatter curve. Dimishing MRS does not tell us much about the behavior of the marginal utilities. It could be that both goods exhibit dimishing marginal utility or that just one does. One thing that we can definitely rule out is that both goods exhibit increasing marginal utility (this would lead to an increasing marginal rate of substitution).

11. Which of the following utility functions exhibit a constant rate of marginal substitution?

$$U_A(x,y) = x + y$$
  $U_B(x,y) = x^2 + y^2$   $U_C(x,y) = x^2 + 2xy + y^2$ 

(a)  $U_A$ .

- (b)  $U_A$  and  $U_B$ .
- (c)  $U_A$  and  $U_C$ .
- (d)  $U_B$  and  $U_C$ .

(c) The most direct way to do this problem is to actually solve for the marginal rate of substitution for each function:

$$MRS_A = -\frac{MU_x}{MU_y} = -\frac{1}{1} = -1$$
$$MRS_B = -\frac{MU_x}{MU_y} = -\frac{2x}{2y} = -\frac{x}{y}$$
$$MRS_C = -\frac{MU_x}{MU_y} = -\frac{2x+2y}{2x+2y} = -1$$

So the marginal rates of substitution for the first and third utility functions are constant. Notice that  $U_C = U_A^2$ , so  $U_C$  is a monotonic transformation of  $U_A$ . Even though the two functions look quite different (and give very different expressions for marginal utility), they actually represent the same preferences.

12. Suppose that your demand for cookies (C) in terms of income (I), the price of cookies  $(p_C)$  and the price of milk  $(p_M)$  is given by:

$$C(I, p_C, p_M) = \frac{40I}{p_C^{\frac{1}{2}} + p_M^{\frac{1}{2}}}$$

Which of the following statements is true?

- (a) Cookies are a normal, ordinary good.
- (b) Cookies are an inferior, ordinary good.
- (c) Cookies are a normal, Giffen good.
- (d) Cookies are an inferior, Giffen good.

(a) From the demand function, we can see that an increase in income will increase C so cookies are a normal good. We can also see that an increase in the price of cookies will make the denominator larger, making C smaller. So cookies are an ordinary good.

- 13. Suppose that meatloaf is a Giffen good and a person only consumes meatloaf and steaks. Both the marginal utility of steaks and the marginal utility of meatloaf are positive. An increase in the price of meatloaf will cause:
  - (a) A decrease in the number of steaks purchased.
  - (b) An increase in the number of steaks purchased.
  - (c) No change in the number of steaks purchased.
  - (d) There is not enough information to tell what will happen to the number of steaks.

(a) If the price of meatloaf goes up, the quantity of meatloaf purchased will increase since it is a Giffen good. So if we're buying more meatloaf at a higher price, our total spending on meatloaf must be higher. This lowers the amount of money left to spend on steaks meaning that we now buy fewer steaks.

- 14. Suppose that the marginal utility of x is always positive and the marginal utility of y is always positive. If a particular consumption bundle  $(x^*, y^*)$  maximizes utility, which of the following must be true?
  - (a) The budget line is tangent to the indifference curve passing through  $(x^*, y^*)$ .
  - (b)  $(x^*, y^*)$  lies on the budget line.
  - (c) (a) and (b).
  - (d) Neither (a) nor (b) must be true.

(b) The bundle must be on the budget line. If it were not, there would be an affordable with more of both goods. Since the marginal utilities of the goods are both positive, this bundle would give higher utility that the original bundle, implying that the original bundle did not actually maximize utility. While the bundle must be on the budget line, it may not be a point of tangency between the budget line and the indifference curve. Cases we have covered in class where it isn't a point of tangency include perfect substitutes and perfect complements.

- 15. Assume pens and pencils are normal, ordinary goods. On a graph with pencils on the horizontal axis and pens on the vertical axis, the steeper the indifference curves are:
  - (a) The more pens a person is willing to trade for an additional pencil.
  - (b) The more pencils a person is willing to trade for an additional pen.
  - (c) The greater the price of pencils is relative to the price of pens.
  - (d) The greater the price of pens is relative to the price of pencils.

(a) A steep indifference curve means that we are willing to give up many units of the good on the vertical axis (pens) for one more unit of the good on the horizontal axis (pencils).

#### SECTION II: SHORT ANSWER (40 points)

For this section, be certain to show your work and clearly label any graphs you draw. Give complete answers but keep them concise. Please place a box around final answers where appropriate.

1. For each part below, use the information given in the graphs to draw a set of three indifference curves on the provided blank graph. Note that the blank graph has X on the horizontal axis and Y on the vertical axis. Be certain to label the direction in which utility is increasing. (6 points each)



The key features here are that the marginal utility is always positive for both goods, telling us that the indifference curves should be downward sloping and utility increases as you move away from the origin, and the marginal utility for both goods is diminishing, implying a diminishing marginal rate of substitution. Diminishing MRS tells us that the indifference curves should be convex.

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There are a few important things to capture with the indifference curves here. First, utility increases as x increases and decreases as y increases telling us that x is a good while y is a bad. This implies upward sloping indifference curves and that utility increase as we move towards more x and less y. Second, both graphs of utility are staight lines, implying constant marginal utility for both goods. This will give us straight indifference curves.

2. Bobby consumes only sandwiches (S) and newspapers (N). His utility function in terms of sandwiches and newspapers is given by:

$$U(S,N) = 10S^{\frac{1}{2}}N^{\frac{1}{2}}$$

(a) Derive expressions for the marginal utility of sandwiches  $(MU_S)$  and the marginal utility of newspapers  $(MU_N)$ . Your expressions should be functions of S and N. You must show your work to get credit (just writing the final expressions will not get credit). (6 points)

To get the expressions for the marginal utilities, we simply need to take the derivative of the utility function with respect to the appropriate good:

$$MU_{S} = \frac{dU(S,N)}{dS}$$
$$MU_{S} = \frac{1}{2}10S^{-\frac{1}{2}}N^{\frac{1}{2}} = 5S^{-\frac{1}{2}}N^{\frac{1}{2}}$$
$$MU_{N} = \frac{dU(S,N)}{dN}$$
$$MU_{N} = \frac{1}{2}10S^{\frac{1}{2}}N^{-\frac{1}{2}} = 5S^{\frac{1}{2}}N^{-\frac{1}{2}}$$

(b) Using your expressions from part (a), derive an expression for the marginal rate of substitution in terms of S and N. Explain whether this utility function exhibits an increasing or diminishing marginal rate of substitution. (8 points)

$$MRS = -\frac{MU_S}{MU_N}$$
$$MRS = -\frac{5S^{-\frac{1}{2}}N^{\frac{1}{2}}}{5S^{\frac{1}{2}}N^{-\frac{1}{2}}} = -\frac{N}{S}$$

The marginal rate of substitution is just minus one times the ratio of the marginal utilities. Notes that you could have set up the ratio with  $MU_S$  in the numerator or  $MU_N$  in the numerator. The important thing is that which ever way you choose has to be consistent with how you set up the ratio of prices in the next part of the question.

From the expression for the MRS, it is clear that the utility function exhibits a diminishing marginal rate of substitution. As we move from left to right along an indifference curve, the value of S will be increasing and the value of N will be decreasing. From the equation for the MRS, we can see that increasing S

and decreasing N both lead to a smaller value for the MRS. So we have a diminishing marginal rate of substitution. (This argument depends on the way in which you set up the MRS. If you had  $MU_N$  in the numerator, it would mean you're looking at a graph with newspapers on the horizontal axis and sandwiches on the vertical axis and you would have to switch S and N in the above explanation).

(c) Find Bobby's optimal number of sandwiches and newspapers if his income is \$60, the price of a sandwich is \$5, and the price of a newspaper is \$1. You must show your work to get credit. (14 points)

To solve for the optimal bundle, we need to use the tangency condition and the budget constraint. We will start with the tangency condition, plugging in the expression for MRS we found in the previous part of the question:

$$MRS = -\frac{p_S}{p_N}$$
$$-\frac{N}{S} = -\frac{p_S}{p_N}$$
$$N = \frac{p_S}{p_N}S$$

Now we can plug this into the budget constraint to solve for a value for S:

$$p_{S}S + p_{N}N = I$$

$$p_{S}S + p_{N}\frac{p_{S}}{p_{N}}S = I$$

$$p_{S}S + p_{S}S = I$$

$$S = \frac{I}{2p_{S}}$$

Now we have a demand equation for S that is only in terms of prices and income, not the value of N. To get a demand function for N we can plug this expression back into our equation for N in terms of S:

$$N = \frac{p_S}{p_N} S$$
$$N = \frac{p_S}{p_N} \frac{I}{2p_S}$$
$$N = \frac{I}{2p_N}$$

With demand functions for S and N, the last step to find the optimal quantities is to simply plug in the given prices and income:

$$S = \frac{I}{2p_S} = \frac{60}{2 \cdot 5} = 6$$
$$N = \frac{I}{2p_N} = \frac{60}{2 \cdot 1} = 30$$

So Bobby's optimal bundle contains six sandwiches and thirty newspapers.