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## Midterm - Solutions

You have until 3:50pm to complete this exam. Be certain to put your name, id number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the short answer questions directly on the exam. You must show your work for full credit. Answers may be left as fractions. Please place a box around final answers when appropriate. Good luck!

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**Name:**

**ID Number:**

**Section:**

### SECTION I: MULTIPLE CHOICE (60 points)

1. Annie's marginal utility from books is positive and her marginal utility from coffee is positive. We can say for certain that:
  - (a) Books and coffee are complements.
  - (b) Annie's indifference curves have a negative slope.
  - (c) Annie's marginal rate of substitution is positive.
  - (d) All of the above.

(b) Since both goods give her positive utility, if Annie received more of one good she would have to receive less of the other good to keep her at the same utility level. This implies downward sloping indifference curves and a negative marginal rate of substitution.
2. Suppose that Bill consumes only water and bread. On a graph with bread on the horizontal axis and water on the vertical axis, the income offer curve has a negative slope. Which of the following is true?
  - (a) Bread and water are both normal goods.
  - (b) Bread and water are both inferior goods.
  - (c) One of the two goods is inferior.
  - (d) Not enough information.

(c) If the income offer curve has a negative slope, when income increases demand for one good must be going up while demand for the other good must be going down. This means that one good is normal while the other good is inferior.
3. If a person's preferences exhibit an increasing marginal rate of substitution, which of the following is definitely not true?
  - (a) The preferences are monotonic.
  - (b) The preferences are convex.
  - (c) The preferences are transitive.
  - (d) The preferences are complete.

(b) With an increasing marginal rate of substitution, the indifference curves will be concave and an average of two bundles on the same indifference curve will lie below that indifference curve. This means that preferences are not convex (extremes are actually preferred to averages in this case).

4. At the current prices for bracelets, an increase in the price of bracelets of 2% would lead to a decrease in demand of 1%. This implies that at the current prices:

- (a) Demand for bracelets is inelastic and revenue would increase if prices were increased by a small amount.
- (b) Demand for bracelets is elastic and revenue would increase if prices were increased by a small amount.
- (c) Demand for bracelets is inelastic and revenue would decrease if prices were increased by a small amount.
- (d) Demand for bracelets is elastic and revenue would decrease if prices were increased by a small amount.

(a) If demand decreases by 1% with a 2% increase in price, the price elasticity of demand is  $-\frac{1}{2}$ . This is between zero and negative one so demand is inelastic. When demand is inelastic, revenue increases with a small price increase.

5. If pollution is a bad, a graph with utility on the vertical axis and pollution on the horizontal axis:

- (a) Will have a negative slope that gets flatter as pollution gets larger.
- (b) Will have a negative slope that gets steeper as pollution gets larger.
- (c) Either (a) or (b) could be true.
- (d) Neither (a) nor (b) could be true.

(c) The fact that pollution is a bad tells us that utility decreases when pollution increases. This gives us a negatively sloped curve on a graph of utility as a function of pollution. We would need to know how the marginal utility is changing to be able to say whether the curve is getting flatter or steeper as pollution increases.

6. Suppose that originally, Starbucks was the only kind of coffee available in town. If several new coffee shops enter the market, we would expect the price elasticity of demand for Starbucks coffee:

- (a) To increase in magnitude.
- (b) To decrease in magnitude.
- (c) To change from positive to negative.
- (d) To change from negative to positive.

(a) The price elasticity of demand will most likely be negative before and after the new coffee shops enter (higher prices would mean lower demand). When more coffee shops enter, it is more likely that customers will switch to other coffee shops when Starbucks increases prices. This larger demand response will lead to a larger magnitude for the elasticity.

7. On a graph with pie on the horizontal axis and punch on the vertical axis, Constance is currently at a bundle on her budget line where her indifference curve is steeper than the budget line. Assuming both pie and punch are good, Constance could increase her utility by:
- Moving up and to the left along her indifference curve.
  - Moving up and to the left along her budget line.
  - Moving down and to the right along her indifference curve.
  - Moving down and to the right along her budget line.
- (d) Moving along the indifference curve would not change her utility. Moving down and to the right along the budget line will move her above her original indifference curve leading to an increase in utility.
8. Which of the following would produce a kinked budget line?
- Giving a person a gift card that can only be spent on one of the two goods she consumes.
  - A change in the price of one good after a certain quantity (for example, bagels cost \$1.50 each for the first five and \$1 for every bagel after that).
  - Both (a) and (b).
  - Neither (a) nor (b).
- (c) The gift card would shift the budget line out but would not change the maximum amount of the other good that could be purchased. This will lead to a new budget line that has a kink at the point where the consumer first starts spending more than the gift card amount on the gift card related good. For answer (b), the change in the price would change the slope of the budget line at the point where the new price kicks in.
9. A Giffen good would have:
- A downward sloping demand curve and an upward sloping Engel curve.
  - A downward sloping demand curve and a downward sloping Engel curve.
  - An upward sloping demand curve and an upward sloping Engel curve.
  - An upward sloping demand curve and a downward sloping Engel curve.
- (d) By definition, demand for a Giffen good increases when price increases giving us an upward sloping demand curve. The only way this is possible is if a Giffen good is also an inferior good, so Giffen goods have a downward sloping Engel curve.
10. Suppose that two utility function,  $U^a(x, y)$  and  $U^b(x, y)$ , represent the same preferences and that  $U^a(x, y) = xy$ . We can say for certain that:
- $U^b(x, y)$  exhibits a diminishing marginal utility for  $x$ .
  - $U^b(x, y)$  exhibits a diminishing marginal rate of substitution.
  - Neither (a) nor (b) is definitely true.
  - Both (a) and (b) are definitely true.
- (b) If the two functions represent the same preferences, they need to give the same marginal rate of substitution at any particular bundle. Notice that the marginal rate of substitution for  $U^a$  is equal to  $-\frac{y}{x}$ . This get smaller as you move down and right along an indifference curve so it is diminishing. Therefore  $U^b$  also exhibits a

diminishing marginal rate of substitution. We can't say any thing about diminishing marginal utility. For example,  $U^b = (U^a)^2$  and  $U^b = (U^a)^{\frac{1}{2}}$  would both give us a new utility function that represents the same preferences as  $U^a$  but the first case would give us increasing marginal utility of  $x$  and  $y$  while the second one would give us decreasing marginal utility of  $x$  and  $y$ .

11. Both chips and salsa have positive marginal utilities. If the demand for chips does not depend on the price of salsa, an increase in the price of salsa will lead to:
  - (a) A positive income effect for chips equal in magnitude to the substitution effect for chips.
  - (b) A negative income effect for chips equal in magnitude to the substitution effect for chips.
  - (c) No income effect and no substitution for chips.
  - (d) None of the above.

(b) There will be a positive substitution effect for chips because chips have become relatively less expensive. If the net change in chips is zero, the income effect must be negative and equal in magnitude to the substitution effect.
12. If Ellen has a downward sloping demand curve and is currently buying a positive quantity of magazines:
  - (a) Her consumer surplus from magazines is exactly zero.
  - (b) Her marginal benefit from the last magazine she purchased is equal to zero.
  - (c) Her marginal benefit from the last magazine she purchased is less than the price of a magazine.
  - (d) Her marginal benefit from the next to last magazine she purchased is greater than the price of a magazine.

(d) Ellen will purchase magazines up to the point where the marginal benefit of the last magazine purchased is equal to the price of the magazine. Therefore the value of the demand curve at any given quantity represents the marginal benefit of that particular magazine. Since the demand curve is downward sloping, the next to last magazine purchased will have a higher marginal benefit than the last magazine purchased and will therefore have a marginal benefit that is greater than the price of magazine.
13. If puppies and kittens are substitutes, then the price offer curve when the price of puppies is varied will be:
  - (a) Downward sloping.
  - (b) Upward sloping.
  - (c) A horizontal line.
  - (d) A vertical line.

(a) As the price of puppies increases, demand for puppies will go down while demand for kittens will go up since kittens are a substitute for puppies. This means that the optimal bundle is moving toward fewer puppies and more kittens as the price of puppies increases, tracing out a downward sloping price offer curve.
14. Fred consumes only pizza and beer and both are ordinary goods. The income elasticity for beer is  $-0.75$ . We can say for certain that:

- (a) Beer is an inferior good.
- (b) Pizza is a normal good.
- (c) Both (a) and (b).
- (d) Neither (a) nor (b).

(c) A negative income elasticity for beer tells us that beer is an inferior good. It also tells us that when income goes up, spending on pizza must go up (spending on beer goes down, if spending on pizza also went down our old bundle couldn't have been an optimal bundle). So pizza is a normal good.

15. Which of the following would lead to a steeper budget line on a graph with food on the horizontal axis and clothing on the vertical axis?

- (a) A change in preferences that increases the marginal utility of food at every level of food.
- (b) A change in preferences that increases the marginal utility of clothing at every level of clothing.
- (c) A change in prices that increases the price of food relative to clothing.
- (d) A change in prices that increases the price of clothing relative to food.

(c) Changes in preferences would affect the slope of the indifference curves but not the budget line. The budget line will get steeper if the price of the good on the horizontal axis (food) increases relative to the price of the good on the vertical axis (clothing).

## SECTION II: SHORT ANSWER (40 points)

1. (15 points) Suppose that you go to an amusement park that has a roller coaster and a ferris wheel. A ride on the roller coaster costs 4 tickets and a ride on the ferris wheel costs 2 tickets. You have a total of 30 tickets. Your utility from roller coaster rides ( $R$ ) and ferris wheel rides ( $F$ ) is given by the following utility function:

$$U(R, F) = 10RF \quad (1)$$

- (a) Write down an appropriate budget equation given the information above. Include actual numbers whenever possible.

Prices are in tickets rather than dollars here so it makes sense to write a budget equation with prices and income all in terms of tickets:

$$p_R R + p_F F = I$$

$$4R + 2F = 30$$

- (b) Derive expressions for the marginal utility of roller coaster rides, the marginal utility of ferris wheel rides, and the marginal rate of substitution.

$$MU_R = \frac{dU(R, F)}{dR} = 10F$$

$$MU_F = \frac{dU(R, F)}{dF} = 10R$$

$$MRS = -\frac{MU_R}{MU_F} = -\frac{10F}{10R} = -\frac{F}{R}$$

Note that you could also have set up the marginal rate of substitution with the marginal utility of ferris wheel rides in the numerator and the marginal utility of roller coaster rides in the denominator. If you did this, you should have gotten  $-\frac{R}{F}$  as your marginal rate of substitution.

- (c) Assuming that you can consume fractions of rides, what is your optimal combination of roller coaster rides and ferris wheel rides?

We first need to set up our tangency condition by setting the slope of the budget line equal to the slope of the indifference curve:

$$-\frac{p_R}{p_F} = -\frac{MU_R}{MU_F}$$

$$-\frac{p_R}{p_F} = -\frac{F}{R}$$

$$F = \frac{p_R}{p_F} R$$

Now we can plug this result into the budget constraint:

$$p_R R + p_F F = I$$

$$p_R R + p_F \frac{p_R}{p_F} R = I$$

$$R = \frac{I}{2p_R}$$

This is our demand equation for  $R$ . To get our demand equation for  $F$ , we can plug this result back into the final equation from our tangency condition:

$$F = \frac{p_R}{p_F} \frac{I}{2p_R} = \frac{I}{2p_F}$$

Plugging in 4 for  $p_R$ , 2 for  $p_F$  and 30 for  $I$  into these two demand equations give us 3.75 for  $R$  and 7.5 for  $F$ .

- (d) If you can only consume integer numbers of rides (no fractions), what is your optimal combination of roller coaster rides and ferris wheel rides? Be certain to show work that justifies your answer.

If we are not allowed to consume fractions of rides, we will need to switch to either 4 roller coaster rides or 3 roller coaster rides. Note that if we choose 4 roller coaster rides, this will leave us with \$14 to spend on ferris wheel rides which would get us 7 rides. If we switch to 3 roller coaster rides, we will be left with \$18 for ferris wheel rides or 9 rides. To see which of these combinations is a better choice, we can simply see which one gives us more utility:

$$U(4, 7) = 10 \cdot 4 \cdot 7 = 280$$

$$U(3, 9) = 10 \cdot 3 \cdot 9 = 270$$

So switching to 4 roller coaster rides and 7 ferris wheel rides gives us more utility than switching to 3 roller coaster rides and 9 ferris wheel rides. If we can't consume fractions of a ride, we will go with 4 roller coaster rides and 7 ferris wheel rides. (Note that we don't need to check all of the affordable combinations of roller coaster rides and ferris wheel rides, just the ones on either side of the the (3.75, 7.5) combination. The reason is that utility is strictly decreasing as we move away from (3.75, 7.5); any affordable combination of  $R$  and  $F$  that has more than 4 roller coaster rides will offer less utility than (4, 7) and any affordable combination with less than 3 roller coaster rides will offer less utility than (3, 9).)

2. (10 points) Suppose that a consumer's demand functions for newspapers ( $N$ ) and magazines ( $M$ ) are:

$$N = \frac{I}{p_N + \frac{p_N^2}{p_M}} \quad (2)$$

$$M = \frac{I}{p_M + \frac{p_M^2}{p_N}} \quad (3)$$

where  $p_N$  is the price of a newspaper,  $p_M$  is the price of a magazine and  $I$  is income.

- (a) Are newspapers and magazines complements, substitutes or neutral goods? Be certain to justify your answer.

To see whether they are complements or substitutes, we need to see how demand for one responds to the price of the other. If we look at the demand function for  $N$ , it gets bigger as  $p_M$  gets larger. So the goods are substitutes. To see this more formally, we can check the sign of the derivative of  $N$  with respect to  $p_M$ :

$$\frac{dN}{dp_M} = -\frac{I}{(p_N + \frac{p_N^2}{p_M})^2} \left(-\frac{p_N^2}{p_M^2}\right) = \left(\frac{p_N}{p_M}\right)^2 \frac{I}{(p_N + \frac{p_N^2}{p_M})^2}$$

Notice that everything in this expression is positive, so the sign of the derivative is positive implying that a change in  $N$  with an increase in  $p_M$  is positive. We could go through the same arguments to show that the change in  $M$  with an increase in  $p_N$  is positive. Either way, we see that the two goods are substitutes.

- (b) Are newspapers an ordinary or Giffen good? Be certain to justify your answer.

Notice that when the price of newspapers increases, the denominator of the demand equation for  $N$  increases, decreasing  $N$ . This implies that newspapers are an ordinary good. A more formal way to confirm this is to check that the derivative of  $N$  with respect to  $p_N$  is negative:

$$\frac{dN}{dp_N} = -\frac{I}{(p_N + \frac{p_N^2}{p_M})^2} \left(1 + 2\frac{p_N}{p_M}\right)$$

Notice that for all positive prices and income, this expression is negative implying that an increase in  $p_N$  leads to decrease in  $N$ . Therefore  $N$  is an ordinary good. (Note that you could have also demonstrated that  $N$  is an ordinary good by showing that  $N$  is a normal good since all normal goods are ordinary goods. Taking the derivative of the demand function with respect to  $I$  is slightly simpler so this approach may seem easier.)

- (c) Assume income is equal to \$100, the original price of a newspaper is \$1 and the original price of a magazine is \$1. Suppose that the price of a newspaper increases to \$3. Calculate the change in demand for magazines due to the income effect and the change in demand for magazines due to the substitution effect.

To calculate the income and substitution effects, we need to calculate the original optimal bundle  $(N_o, M_o)$ , the final optimal bundle  $(N_f, M_f)$  and the intermediate bundle,  $(N_i, M_i)$  where we hold adjusted income constant but allow the



prices to change. The original bundle is the optimal bundle under the original prices and income:

$$N_o = \frac{100}{1 + \frac{1^2}{1}} = 50$$

$$M_o = \frac{100}{1 + \frac{1^2}{1}} = 50$$

The final bundle is the optimal bundle under the new prices and actual income:

$$N_f = \frac{100}{3 + \frac{3^2}{1}} = 8.33$$

$$M_f = \frac{100}{1 + \frac{1^2}{3}} = 75$$

To get the intermediate bundle, we first need to find the adjusted income that makes the original bundle affordable under the new prices:

$$\tilde{I} = 3N_o + 1M_o = 3 \cdot 50 + 1 \cdot 50 = 200$$

Now we can get the intermediate bundle by plugging this adjusted income and the new prices into the demand functions:

$$N_i = \frac{200}{3 + \frac{3^2}{1}} = 16.67$$

$$M_i = \frac{200}{1 + \frac{1^2}{3}} = 150$$

Now we can calculate our income and substitution effects for magazines:

$$\text{sub effect} = M_i - M_o = 150 - 50 = 100$$

$$\text{inc effect} = M_f - M_i = 75 - 150 = -75$$

3. (15 points) There are two consumers in the market for sandwiches, Greg and Harry. The inverse demand curves for Greg and Harry are given by:

$$p(S_g) = 100 - \frac{1}{2}S_g \quad (4)$$

$$p(S_h) = 50 - S_h \quad (5)$$

where  $S_g$  is the number of sandwiches demanded by Greg and  $S_h$  is the number of sandwiches demanded by Harry.

- (a) Graph the market demand curve for sandwiches labeling all slopes, intercepts and kinks with their numerical values.

Notice from Greg's demand function that when  $S_g$  equals zero, price is equal to 100. For Harry, price equals 50 when demand hits zero. This tells us that our market demand function will hit the vertical axis at a price of 100 when Greg enters the market and have a kink at 50 when Harry enters the market. To get equations for the two segments of the market demand curve, we first need to rearrange the inverse demand functions to be quantity as a function of price:

$$S_g = 200 - 2p$$

$$S_h = 50 - p$$

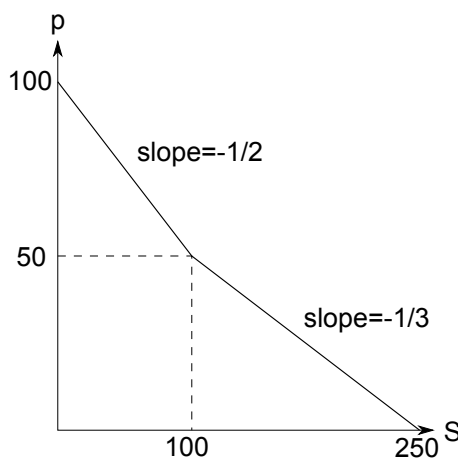
Now we can get our market demand curve:

$$S_{market} = \begin{cases} 0 & \text{for } p > 100 \\ S_g & \text{for } 50 < p < 100 \\ S_g + S_h & \text{for } p < 50 \end{cases}$$

$$S_{market} = \begin{cases} 0 & \text{for } p > 100 \\ 200 - 2p & \text{for } 50 < p < 100 \\ 200 - 2p + 50 - p & \text{for } p < 50 \end{cases}$$

$$S_{market} = \begin{cases} 0 & \text{for } p > 100 \\ 200 - 2p & \text{for } 50 < p < 100 \\ 250 - 3p & \text{for } p < 50 \end{cases}$$

This market demand equation is graphed below:



- (b) Suppose that both Greg and Harry consider sandwiches and potato chips complements. If the price of potato chips increases, how would you expect the graph you made in part (a) to change?

If potato chips get more expensive and sandwiches and potato chips are complements, demand for sandwiches will go down at any given price of sandwiches. This means that at any given price on the vertical axis of the graph, the quantity of sandwiches demanded would be to the left of the original demand curve. Therefore the entire demand curve is shifting down and left.

- (c) At what price would a small increase in the price of sandwich have no effect on the sandwich shop's revenue? (Assume the sandwich shop is operating in a price range where both Greg and Harry are buying positive quantities of sandwiches.)

An increase in price will have no effect on revenue when demand is unit elastic. We simply need to find the price at which elasticity is equal to  $-1$ . We start with the basic elasticity formula:

$$\varepsilon = \frac{p}{S} \frac{dS}{dp}$$

We are told that we are on the segment of the demand curve where both customers are buying sandwiches. Plugging in the equation for this section of the demand curve for  $S$  and taking the derivative to get  $\frac{dS}{dp}$  gives us:

$$\begin{aligned} \varepsilon &= \frac{p}{250 - 3p} \frac{dS}{dp} \\ \varepsilon &= \frac{p}{250 - 3p} (-3) \\ \varepsilon &= -\frac{3p}{250 - 3p} \end{aligned}$$

Now we can plug in  $-1$  for  $\varepsilon$  and solve for  $p$ :

$$-1 = -\frac{3p}{250 - 3p}$$

$$250 - 3p = 3p$$

$$250 = 6p$$

$$p = 41.66$$