
Midterm 1 - Solutions

You have until 11:50am to complete this exam. Be certain to put your name, ID number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the long answer questions directly on the exam. You must show your work where relevant for full credit. Good luck!

Name:

ID Number:

Section:

SECTION I: MULTIPLE CHOICE (60 points)

1. Suppose that Adam is indifferent between getting a bundle with four magazines and two books and a bundle with two magazines and four books. If Adam prefers a bundle with four magazines and two books to one with three magazines and three books, we could conclude that:
 - (a) Adam's preferences are not monotonic.
 - (b) Adam's preferences are not transitive.
 - (c) Adam's preferences are not complete.
 - (d) Adam's preferences are not convex.

(d) If Adam's preferences were convex and he is indifferent between two bundles, he should prefer an average of those two bundles. In this case, he is indifferent between (4 magazines, 2 books) and (2 magazines, 4 books). If his preferences were convex, he should prefer (3 magazines, 3 books) to either of these bundles.
2. On a graph with apples on the horizontal axis and oranges on the vertical axis, Bob's indifference curves are upward sloping. Which of the following statements is true?
 - (a) The marginal utility of apples and the marginal utility of oranges are both positive.
 - (b) Apples and oranges are both bads.
 - (c) The marginal utilities of apples and oranges have opposite signs.
 - (d) None of the above.

(c) If his indifference curves are upward sloping, it tells us that an increase in apples has the opposite effect on utility as an increase in oranges since an increase in apples is needed to offset the change in utility from an increase in oranges to stay on the same indifference curve. This means that the two goods must have opposite signs for their marginal utilities.
3. If two utility functions represent the same preferences, they will both lead to:
 - (a) The same functions for the marginal utility of each good.
 - (b) The same function for the marginal rate of substitution.

- (c) The same value for the utility of a specific bundle.
- (d) All of the above.

(b) They must produce the same marginal rate of substitution. The *MRS* tells us how the consumer is willing to trade between the two goods. If the two functions produced different expressions for the *MRS* it would mean the consumer would trade between the two goods differently under one utility function than other the other utility function. Clearly this would mean preferences differ between the two functions. It is not necessary for the marginal utilities or the levels of utility to be the same for both functions.

4. Which of the following would definitely make the set of affordable bundles larger?

- (a) Increasing the price of one good and decreasing the price of the other.
- (b) Decreasing the price of one good but leaving the price of the other good unchanged.
- (c) Increasing income and increasing the prices of both goods.
- (d) Decreasing income and decreasing the prices of both good.

(b) Decreasing the price of one good will shift the intercept for that good out to a larger value. This rotation of the budget line will mean that the new set of affordable bundles will contain the entire old set of affordable bundles but will also contain new bundles that weren't previously affordable.

5. Suppose that both candy and cigarettes are goods (rather than bads) and both exhibit diminishing marginal utility. Which of the following graphs would have a curve with a negative slope?

- (a) A graph of the marginal utility of cigarettes as a function of the number of cigarettes with marginal utility on the vertical axis cigarettes on the horizontal axis.
- (b) A graph of an indifference curve with a candy on the vertical axis and cigarettes on the horizontal axis.
- (c) Neither (a) nor (b).
- (d) Both (a) and (b).

(d) Marginal utility is a decreasing function of the number of cigarettes, so its graph will be downward sloping. The indifference curves for two goods (rather than a good and a bad) will always be downward sloping.

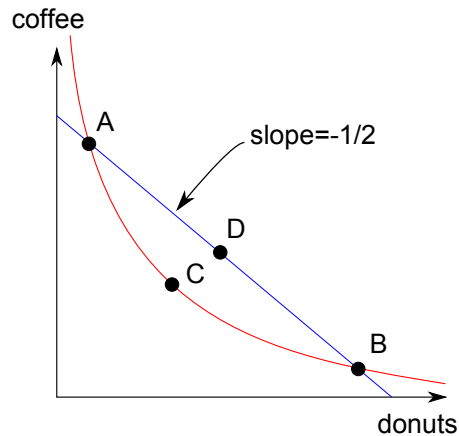
6. Suppose goods x and y are the only two goods Carla can buy. Assuming Carla always maximizes her utility, when would she choose to not spend all of her money?

- (a) When x and y are perfect substitutes.
- (b) When one of the goods is a bad.
- (c) When her marginal rate of substitution is increasing.
- (d) When both of the goods are bads.

(d) If both goods are bad, her utility will be maximized by buying nothing (anything she buys would lower her utility).

Use the figure below to answer questions 7 through 9. The horizontal axis measures the number of donuts in a consumption bundle. The vertical axis measures the number of cups

of coffee in a consumption bundle. The graph depicts David's budget line and his indifference curve for a utility level of 200 units. The marginal utility of coffee and the marginal utility of donuts are both positive.



7. Suppose that at his current bundle, the marginal utility of coffee is twice the marginal utility of donuts and David is spending all of his money. Which of the following bundles could David possibly be at?
- (a) Bundle A.
 - (b) Bundle B.
 - (c) Bundle C.
 - (d) None of the above.

(d) Note that the magnitude of the marginal rate of substitution is equal to the marginal utility of donuts divided by the marginal utility of coffee, which is equal to $\frac{1}{2}$ (since the marginal utility of coffee is twice as big as the marginal utility of donuts). So at his current bundle, the slope of the indifference curve (the *MRS*) is equal to the slope of the budget line. This rules out bundles *A* and *B*. Since he is spending all of his money, we can rule out bundle *C*.

8. Which of the following can we say for certain?
- (a) The utility David gets from bundle A is greater than the utility he gets from bundle C.
 - (b) The utility David gets from bundle C is less than 200 units.
 - (c) The utility David gets from bundle D is greater than 200 units.
 - (d) None of the above.

(c) Note that bundle *C* is on the indifference curve for a utility level of 200, so it must give David 200 units of utility. Bundle *D* has more of both goods in it and since the marginal utilities of both goods are positive, it must give David more utility than bundle *C*.

9. Assuming all of David's indifference curves have a shape similar to the one on the graph, which of the following statements is definitely true?

- (a) When moving from left to right along an indifference curve, the marginal utility of donuts is increasing.
- (b) When moving from left to right along an indifference curve, the marginal utility of donuts is decreasing.
- (c) When moving from left to right along an indifference curve, the ratio of the marginal utility of donuts to the marginal utility of coffee is increasing.
- (d) When moving from left to right along an indifference curve, the ratio of the marginal utility of donuts to the marginal utility of coffee is decreasing.
- (d) Notice that the indifference curve gets flatter from left to right. The magnitude of the slope of the indifference curve is the ratio of the marginal utility of donuts to the marginal utility of coffee. Since the curve is getting flatter, this ratio must be getting smaller.
10. On a graph with shirts on the vertical axis and shoes on the horizontal axis, which of the following would make the vertical intercept of the budget line larger?
- (a) An increase in income.
- (b) An increase in the price of shirts.
- (c) A decrease in the price of shoes.
- (d) None of the above.
- (a) An increase in income would shift both of the intercepts out to larger values.
11. Suppose that Elizabeth consumes only goods x and y and her marginal utility from x and marginal utility from y are given by:

$$MU_x = 2y \tag{1}$$

$$MU_y = 4x \tag{2}$$

If the price of y is \$2 and the price of x is \$4, Elizabeth's optimal bundle will contain:

- (a) Twice as much x as y .
- (b) Twice as much y as x .
- (c) Four times as much x as y .
- (d) Four times as much y as x .
- (d) At Elizabeth's optimal bundle, the slope of the budget line will equal the slope of the indifference curve:
- $$-\frac{p_x}{p_y} = -\frac{MU_x}{MU_y}$$
- $$\frac{4}{2} = \frac{2y}{4x}$$
- $$4x = y$$
- So there will be the amount of y will be four times larger than the amount of x .
12. Suppose that Frank prefers the bundle (x_a, y_a) to the bundle (x_b, y_b) and prefers the bundle (x_b, y_b) to the bundle (x_c, y_c) . Which of the following statements is true?

- (a) If Frank's preferences are convex, we can say for certain that he prefers the bundle (x_a, y_a) to the bundle (x_c, y_c) .
- (b) If Frank's preferences are monotonic, we can say for certain that x_a is larger than x_b .
- (c) If Frank's preferences are transitive, we can say for certain that he prefers the bundle (x_a, y_a) to the bundle (x_c, y_c) .
- (d) If Frank's preferences are complete, we can say for certain that x_a is larger than x_b .
- (c) This follows directly from the definition of transitivity.
13. Suppose that Gary's consumption bundles consist of only cups of water and handfuls of sand. The marginal utility of a cup of water is always positive but diminishing as the number of cups increases. The marginal utility of a handful of sand is always zero. On a graph with cups of water on the vertical axis and handfuls of sand on the horizontal axis, Gary's indifference curves will be:
- (a) Vertical lines.
- (b) Downward sloping curves that get flatter from left to right.
- (c) Upward sloping curves that get flatter from left to right.
- (d) Horizontal lines.
- (d) Increasing or decreasing the number of handfuls of sand has no effect on utility. So if you hold cups of water constant and change the amount of sand, you would stay on the same indifference curve. This would trace out a horizontal line.
14. Suppose that the price of a pen is always \$2. The price of a pencil is \$2 for each of the first 10 pencils a person buys. After that, each additional pencil costs \$1. On a graph with pencils on the vertical axis and pens on the horizontal axis, the budget line will:
- (a) Have a kink at 10 pens and will be steeper to the right of the kink than to the left of the kink.
- (b) Have a kink at 10 pens and will be flatter to the right of the kink than to the left of the kink.
- (c) Have a kink at 10 pencils and will be steeper to the right of the kink than to the left of the kink.
- (d) Have a kink at 10 pencils and will be flatter to the right of the kink than to the left of the kink.
- (d) If pencils are on the vertical axis, the magnitude of the slope of the budget line is equal to the price of pens divided by the price of pencils. When you get above 10 pencils, the denominator of this ratio gets smaller making the slope steeper. So the upper portion of the budget line will have a steeper slope than the lower portion.
15. Suppose that Harry always chooses the consumption bundle of apples (A) and oranges (O) that maximizes his utility. He likes both apples and oranges. His marginal rate of substitution is given by:

$$MRS = -\frac{MU_A}{MU_O} = -2\frac{O}{A} \quad (3)$$

If the price of apples increases:

- (a) Harry will increase the number of apples he consumes.

- (b) Harry will move to a bundle on a lower indifference curve than his bundle before the price change.
- (c) Harry will have a higher utility level than before the price change.
- (d) None of the above.

(b) If he was maximizing utility before the price change, Harry would have been at a bundle where his indifference curve was tangent to the budget line. The price change will rotate the budget line in. The new budget line will be completely below the old indifference curve. So no matter where Harry's new optimal bundle is on this new budget line, it will be below his old indifference curve.

SECTION II: SHORT ANSWER (40 points)

1. (15 points) Julius always consumes hot dogs (H) and buns (B) in a one to one ratio. He gets no additional utility from an extra hot dog unless he also gets an extra bun. Likewise, he gets no additional utility from an extra bun unless he also gets an extra hot dog.

(a) Write down a utility function that represents Julius's preferences.

This is an example of perfect complements: hot dogs and buns are always consumed in a one to one ratio. The utility function for perfect complements will take the form of a min function:

$$U(H, B) = \min(H, B)$$

The coefficients in front of H and B should be equal since the two goods are consumed in equal proportions.

- (b) Suppose that the price of a hot dog is \$2 and the price of a bun is \$1. Julius has \$21 to spend. Sketch a graph with buns on the horizontal axis and hot dogs on the vertical axis that shows all of the following: Julius's budget line, Julius's optimal bundle, the indifference curve passing through Julius's optimal bundle. Be certain to label the endpoints of the budget line and the optimal bundle with their numerical values.

Notice that we cannot use the standard approach of setting the ratio of prices equal to the ratio of the marginal utilities. The problem is that we cannot calculate the marginal utilities. However, we can use a little bit of logic to arrive at the answer. Julius always consumes one hot dog for every bun. If the number of hot dogs was greater than the number of buns, those extra hot dogs aren't giving Julius any extra utility and the money spent on them is wasted. He should take some of that money and use it to increase the number of buns. The same argument holds if the number of buns was greater than the number of hot dogs. So the only way Julius is getting the most for his money is if the number of hot dogs equals the number of buns:

$$H = B$$

Plugging this into the budget constraint will allow us to find the optimal number of hot dogs and buns in terms of income and prices:

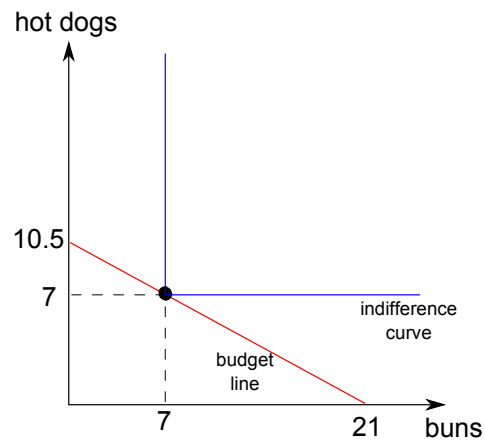
$$p_H H + p_B B = I$$

$$p_H B + p_B B = I$$

$$B = \frac{I}{p_H + p_B}$$

$$B = \frac{21}{2 + 1} = 7$$

So Julius will buy 7 buns and 7 hot dogs (since the number of hot dogs will equal the number of buns). This is shown on the graph below:



The endpoints of the budget line can be found by dividing income by the price of the good in question. The slope of the budget line is the ratio of the prices ($-\frac{p_H}{p_B}$ or $-\frac{1}{2}$ if buns are on the horizontal axis). The indifference curve should be L-shaped. The corner of this L-shape should just touch the budget line at the optimal bundle.

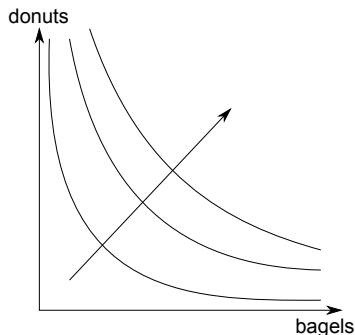
2. (15 points) For each scenario below, graph a set of three indifference curves consistent with the description of the person's preferences for the two goods. Be certain to clearly label the axes and to include an arrow showing the direction of increasing utility on each graph.

- (a) Kristin likes both bagels and donuts. Every additional bagel Kristin gets increases her utility by 10 units. Every additional donut Kristin gets increases her utility but by an amount less than the previous one did. Your graph should have bagels on the horizontal axis and donuts on the vertical axis.

First, note that both bagels and donuts are good, so the indifference curves will be downward sloping. The slope of the indifference curves will be given by the marginal rate of substitution:

$$MRS = -\frac{MU_B}{MU_D}$$

Notice that as you move down and right along an indifference curve, MU_D will be getting larger (since the marginal utility of donuts is diminishing as D gets bigger) and MU_B will be staying the same. This means that the magnitude of the MRS will be getting smaller making the indifference curves flatter as you move from left to right. So your graph should depict downward sloping, convex indifference curves. Since both goods are good, the direction of increasing utility is away from the origin.



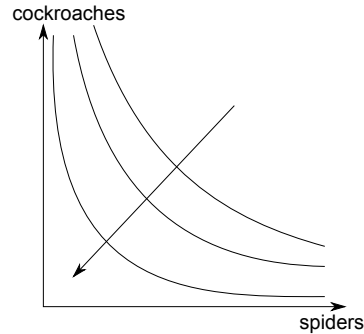
- (b) Larry is scared of both spiders and cockroaches. Every extra spider lowers Larry's utility but but a smaller amount than the previous one did. Every extra cockroach lowers Larry's utility but by a smaller amount than the previous one did. Your graph should have spiders on the horizontal axis and cockroaches on the vertical axis.

Both goods are bads, so the indifference curves will be downward sloping (if you give Larry more spiders you have to compensate him by taking away some cockroaches). The marginal rate of substitution for Larry will be:

$$MRS = -\frac{MU_S}{MU_C}$$

As you move down and to the right along an indifference curve, MU_S will be getting smaller since it decreases as S increases and MU_C will be getting larger since it increases as C gets smaller. So the magnitude of the MRS will be getting smaller going from left to right giving us convex indifference curves. So

just like part (a), we have downward sloping, convex indifference curves. The difference is that now the direction of increasing utility is toward to origin (Larry would prefer to have as few spiders and cockroaches as possible).



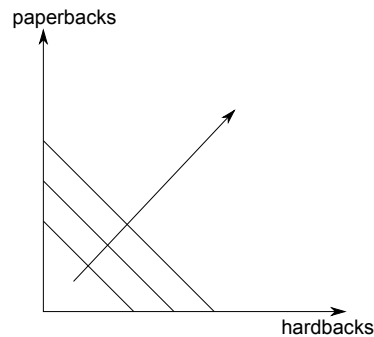
- (c) Mark likes books. He gets the same amount of enjoyment out of a paperback as he does out of a hardback no matter how many of each he already has. Your graph should have hardbacks on the horizontal axis and paperbacks on the vertical axis.

Both goods are good so we have downward sloping indifference curves. If Mark always gets the same enjoyment out of a paperback as he does out of a hardback, we can say that the marginal utility from a paperback equals the marginal utility from a hardback. So his marginal rate of substitution is:

$$MRS = -\frac{MU_H}{MU_P}$$

$$MRS = -1$$

So the indifference curves will be straight lines with a slope of negative one. Since both goods are good, the direction of increasing utility will be away from the origin.



3. (10 points) Suppose that Nancy's utility from CDs (C) and books (B) is given by the following utility function:

$$U(C, B) = C^2 B^3 \quad (4)$$

- (a) Do Nancy's preferences exhibit a diminishing marginal utility for books? Be certain to justify your answer.

The marginal utility of books is given by:

$$MU_B = \frac{dU}{dB} = 3C^2 B^2$$

Notice that this expression gets larger as B gets larger, so the marginal utility of books is increasing, not diminishing.

- (b) Do Nancy's preferences exhibit a diminishing marginal rate of substitution? Be certain to justify your answer.

The marginal rate of substitution corresponding to a graph with books on the horizontal axis and CDs on the vertical axis is given by:

$$MRS = -\frac{MU_B}{MU_C}$$

$$MRS = -\frac{\frac{dU}{dB}}{\frac{dU}{dC}}$$

$$MRS = -\frac{3C^2 B^2}{2CB^3}$$

$$MRS = -\frac{3C}{2B}$$

As you move down and right along an indifference curve, making B larger and C smaller, the magnitude of the MRS is getting smaller so Nancy's preferences do exhibit a diminishing marginal rate of substitution. Note that you would have also arrived at the same conclusion if you had thought of a graph with CDs on the horizontal axis, giving you an MRS equal to $-\frac{2B}{3C}$ that gets smaller in magnitude as the number of CDs increase and the number of books decreases when moving down and right along an indifference curve.