
Midterm 1 - Solutions

You have until 1:00pm to complete this exam. Be certain to put your name, id number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the long answer questions directly on the exam. You must show your work for full credit. Answers may be left as fractions. Please place a box around final answers when appropriate. Good luck!

Name:

ID Number:

Section:

SECTION I: MULTIPLE CHOICE (60 points)

- Suppose apples and bananas are perfect substitutes. On a graph with apples on the horizontal axis and bananas on the vertical axis, if the price of bananas doubles:
 - The indifference curves will get steeper.
 - The budget line will get steeper.
 - The indifference curves will get flatter.
 - The budget line will get flatter.

(d) If the price of bananas goes up, the slope of the budget line ($-\frac{p_a}{p_b}$) will get smaller in magnitude, meaning a flatter budget line.
- Ansel likes dogs more than he likes cats, likes cats more than he likes snakes and likes snakes more than he likes dogs. We can say for certain that Ansel's preferences:
 - Are not monotonic.
 - Are not convex.
 - Are not transitive.
 - Are not complete.

(c) If his preferences were transitive, Ansel liking dogs more than cats and cats more than snakes would imply that he likes dogs more than snakes. Since this is not true, his preferences cannot be transitive.
- Suppose that an ice cream store gives you your first ice cream cone for free. After that, each ice cream cone costs \$1. Cookies cost \$2. If you consume only ice cream cones and cookies, the budget line on a graph with ice cream cones on the horizontal axis and cookies on the vertical axis will:
 - Have one segment that is a vertical line and one segment that has a slope of -.5.
 - Have one segment that is a horizontal line and one segment that has a slope of -2.
 - Have one segment that is a horizontal line and one segment that has a slope of -.5.
 - Have one segment that is a vertical line and and one segment that has a slope of -2.

(c) The budget line will have a horizontal segment for the upper left portion of the budget line (along this segment, the person is spending all of his money on cookies but can also consume anywhere between 0 and 1 ice cream cones because the first cone is free). The rest of the budget line will be a standard, downward sloping budget line with a slope given by $-\frac{p_I}{p_C}$ which is $-\frac{1}{2}$.

4. The optimal quantity of chips is given by $C = \frac{I}{2p_c + 4p_s}$ where I is income, p_c is the price of a bag of chips and p_s is the price of a can of soda. If the price of a can of soda goes up:

- (a) The Engel curve for chips will get steeper.
- (b) The Engel curve for chips will get flatter.
- (c) The Engel curve for chips will not change.
- (d) None of the above.

(a) From the demand equation, we can see that the slope of the Engel curve (I on the vertical axis, C on the horizontal axis) will be $2p_c + 4p_s$. If p_s gets larger, this slope gets larger.

5. Suppose that Betty has well behaved convex preferences and is currently consuming a bundle that costs her entire income. At her current bundle, the marginal utility of x is twice the marginal utility of y . The price of x is \$4 and the price of y is \$1. Which of the following statements is true?

- (a) Betty is currently maximizing her utility given her budget constraint.
- (b) Betty could increase her utility by moving down and to the right along her budget line.
- (c) Betty could increase her utility by moving up and to the left along her budget line.
- (d) Betty could increase her utility by moving down and to the right along her indifference curve.

(c) Betty is currently at a point where the budget line (with a slope of $-\frac{p_x}{p_y}$ or -4) is steeper than the current indifference curve (with a slope of $-\frac{MU_x}{MU_y}$ or -2). This means that if she moves up and to the left along the budget line, she can reach a higher indifference curve.

6. Which of the following utility functions represents preferences for which extremes are preferred to averages?

- (a) $U(x, y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$.
- (b) $U(x, y) = \frac{1}{2}x^3 + y$.
- (c) $U(x, y) = 3x^{\frac{1}{2}} + 3y^{\frac{1}{2}}$.
- (d) None of the above.

(b) The second utility function is the only one for which the marginal rate of substitution ($MRS = -\frac{3}{2}x^2$) is increasing in magnitude as x gets larger. This implies concave indifference curves. An average of any two bundles on the same indifference curve would lie below that indifference curve.

7. Which of the following changes would increase the set of affordable bundles?

- (a) Doubling the prices of both goods.

- (b) Doubling the prices of both goods and doubling income.
- (c) Doubling the price of one good and doubling income.
- (d) None of the above.

(c) Let's say that it was the price of x that doubles. If we spend all of our money on x , we can now buy $\frac{2I}{2p_x}$ units, the same amount as before. If we spend all of our money on y (for which the price stayed the same) we could buy $\frac{2I}{p_y}$, twice as many units as before. So our budget line is rotating out, increasing the set of affordable bundles.

8. Suppose that pollution and mosquitoes are both bads. The slope of an indifference curve on a graph with mosquitoes on the horizontal axis and pollution on the vertical axis:

- (a) Will be positive.
- (b) Will be negative.
- (c) Be a vertical line.
- (d) Be a horizontal line.

(b) If you increase the amount of pollution, this will make a person worse off. To bring them back to their original utility level, you would need to decrease the number of mosquitoes. This gives us downward sloping indifference curves just like we would get with two goods. The difference is that the direction of increasing utility is toward the origin.

9. Suppose x and y are both goods. On a graph with x on the horizontal axis and y on the vertical axis, you are currently at a bundle on the budget line (with positive quantities of both goods) where the slope of the indifference curve is steeper than the slope of the budget line. You can increase utility by:

- (a) Moving up and to the left along the budget line.
- (b) Moving down and to the right along the budget line.
- (c) Moving down and to the left.
- (d) You are already maximizing utility.

(b) Moving down and to the right along the budget line will move you above your current indifference curve, increasing your utility.

10. Coffee and milk are complements (but not perfect complements). Increasing either coffee or milk increases utility. The slope of the price offer curve when the price of coffee is varied will be:

- (a) Positive for a graph with coffee on the horizontal axis and milk is on the vertical axis.
- (b) Positive for a graph with milk on the horizontal axis and coffee on the vertical axis.
- (c) Neither (a) nor (b).
- (d) Both (a) and (b).

(d) As coffee gets cheaper, we will buy more coffee because more coffee increases utility and we will buy more milk because milk is a complement. This means that each time the price of coffee is lowered, we move to a bundle with more of both goods. This will trace out a price offer curve with a positive slope regardless of which good is on which axis.

11. Suppose that the marginal utility from candy is always positive but diminishing as the amount of candy increases. The marginal utility from vegetables is constant and negative. Which of the following statements is true?
- (a) Utility will be maximized by spending all of a person's income on vegetables.
 - (b) The utility maximizing bundle will not use all of a person's income.
 - (c) The utility maximizing bundle will contain zero vegetables.
 - (d) None of the above.
- (c) Candy always gives positive utility while vegetables always give negative utility. It would never make sense to spend any money on vegetables since that money could be spent on candy. Since more candy is always better, it makes sense to spend your full income on candy.
12. If two bundles give a consumer the same utility:
- (a) They must be on the same indifference curve.
 - (b) They must cost the same amount of money.
 - (c) Both (a) and (b).
 - (d) Neither (a) nor (b).
- (a) By definition, all bundles on the same indifference curve give the consumer the same level of utility. However, knowing that two bundles are on the same indifference curve tells us nothing about how much the bundles cost.
13. Suppose that Chuck's utility maximizing bundle has 5 units of x and 10 units of y in it. If a bundle with 8 units of x and 8 units of y would give Chuck a higher level of utility, it must:
- (a) Lie below the budget line.
 - (b) Lie above the budget line.
 - (c) Lie on a lower indifference curve than Chuck's utility maximizing bundle.
 - (d) None of the above.
- (b) If it were on or below Chuck's budget line, he could have chosen that bundle and increased his utility. That would imply that $(5, 10)$ could not have been the utility maximizing bundle. It must be the case that the bundle $(8, 8)$ is unaffordable, meaning that it is above Chuck's budget line.
14. If we know that a person's preferences for salt and pepper are monotonic, we know that her indifference curves will be:
- (a) Convex.
 - (b) Concave.
 - (c) Upward sloping.
 - (d) Downward sloping.
- (d) If preferences are monotonic, then all bundles with both more salt and more pepper than the current bundle would be on a higher indifference curve and all bundles with less salt and less pepper than the current bundle would be on a lower indifference curve. This rules out the possibility that the indifference curve passing through the bundle is upward sloping. It must be downward sloping.

15. Suppose that extra hot dogs always make us happier but the change in utility from an additional hot dog gets smaller as the number of hot dogs gets larger. A graph of utility as a function of hot dogs will have a _____ slope and a graph of marginal utility as a function of hot dogs will have a _____ slope.
- (a) Positive, positive.
 - (b) Positive, negative.
 - (c) Negative, positive.
 - (d) Negative, negative.

(b) If more hotdogs increase utility, then utility as a function of hotdogs must be positively sloped. However, we are told that the marginal utility is decreasing as the number of hotdogs increases implying that the graph of utility gets flatter as hotdogs increase and that a graph of marginal utility as a function of hotdogs will be negatively sloped.

SECTION II: SHORT ANSWER (40 points)

1. (20 points) Suppose that you have ten hours a week to watch TV. You can either watch comedies or dramas. Your utility from the number of comedies you watch (C) and the number of dramas you watch (D) is given by:

$$U(C, D) = C^{\frac{1}{2}} D^{\frac{1}{2}} \quad (1)$$

Comedies are half an hour long and dramas are one hour long.

- (a) Write down an equation giving your budget constraint in terms of the number of comedies you watch (C) and the number of dramas you watch (D). The only variables left in your equation should be C and D , everything else should be numbers. (Hint: Your budget will not be in terms of dollars.)

You are constrained by the amount of time you have available which is ten hours. Your budget constraint is simply that the time spent watching comedies and the time spent watching dramas cannot exceed your total time available. Writing out this budget constraint in units of hours gives us:

$$\frac{1}{2}C + 1 \cdot D = 10$$

You could also have written this as $\frac{1}{2}C + D \leq 10$ but since both C and D are good, we will definitely spend all of our time. Note that our prices here are $\frac{1}{2}$ for comedies (since one comedy costs us one half hour of time) and 1 for dramas (since one drama costs us one hour of time).

- (b) Derive expressions for the marginal utility of comedies (MU_C), the marginal utility of dramas (MU_D) and the marginal rate of substitution (MRS).

$$MU_C = \frac{dU(C, D)}{dC} = \frac{1}{2}C^{-\frac{1}{2}} D^{\frac{1}{2}}$$

$$MU_D = \frac{dU(C, D)}{dD} = \frac{1}{2}C^{\frac{1}{2}} D^{-\frac{1}{2}}$$

$$MRS = -\frac{MU_C}{MU_D} = -\frac{\frac{1}{2}C^{-\frac{1}{2}} D^{\frac{1}{2}}}{\frac{1}{2}C^{\frac{1}{2}} D^{-\frac{1}{2}}} = -\frac{D}{C}$$

Note that you could have set up your marginal rate of substitution with MU_D in the numerator and MU_C in the denominator. This have led to an MRS of $-\frac{C}{D}$. The way in which you set up your marginal rate of substitution here determines the way you need to set up the ratio of prices when solving for the optimal bundle in the next part.

- (c) Find the optimal number of comedies. Assume that you can choose to watch fractions of comedies and dramas (for example, you could decide to watch 9.3 comedies, you wouldn't have to watch either 9 or 10).

Based on the utility function, we will have well-behaved convex indifference curves. This means that we can follow our approach of using the tangency condition and the budget constraint to solve for the optimal bundle:

$$-\frac{p_C}{p_D} = MRS$$

$$-\frac{p_C}{p_D} = -\frac{D}{C}$$

$$D = \frac{p_C}{p_D} C$$

$$p_C C + p_D D = I$$

$$p_C C + p_D \frac{p_C}{p_D} C = I$$

$$2p_C C = I$$

$$C = \frac{I}{2p_C}$$

Plugging in the appropriate value for income (ten hours) and for the price of comedies (one half hour per comedy), we can get our final answer:

$$C = \frac{10}{2 \cdot \frac{1}{2}}$$

$$C = 10$$

2. (20 points) Suppose that your utility from cups of coffee (C) and cups of tea (T) is given by:

$$U(C, T) = 10C + 5T \quad (2)$$

- (a) How many cups of coffee are you willing to trade for one cup of tea?

From the utility function, we can see that the marginal utility of coffee is 10 and the marginal utility of tea is 5. So one additional cup of coffee offers twice the utility of one additional cup of tea. Given these marginal utilities, you would be willing to trade one cup of coffee for two cups of tea, or one half a cup of coffee for one cup of tea.

- (b) If the price of a cup of coffee is \$1, the price of a cup of tea is \$1 and your income is \$20, how many cups of coffee will you buy?

One dollar spent on coffee would get you one cup which would increase your utility by ten units ($MU_C = 10$). That same dollar spent on tea would get you one cup which would only increase your utility by five units ($MU_T = 5$). Clearly you are better off spending that dollar on coffee. This is true regardless of how many cups of coffee you already have. So all of your money will be spent on coffee and none of it will be spent on tea. This leads to the following optimal quantities of coffee and tea:

$$C = \frac{I}{p_C} = \frac{20}{1} = 20$$

$$T = 0$$

- (c) Graph the Engel curve for coffee and the Engel curve for tea when the price of a cup of coffee is \$1 and the price of a cup of tea is \$1. Use a separate graph for each Engel curve. Label intercepts and slopes with numerical values where possible.

Using the same reasoning as above, we would have the following demand equations for coffee and tea under the current prices:

$$C = \frac{I}{p_C} = \frac{I}{1} = I$$

$$T = 0$$

The demand equation for coffee shows us that the Engel curve will be a straight line passing through the origin with a slope of one. The demand equation for tea show us that the Engel curve will be a vertical line at $T = 0$ (as income increases, the optimal quantity of tea stays at zero). These Engel curves are shown below:

