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## Midterm 1 - Solutions

You have until 4:30pm to complete this exam. Be certain to put your name, id number and section on both the exam and your scantron sheet and fill in test form A on the scantron. Answer all multiple choice questions on your scantron sheet. Choose the single best answer for each question; if you fill in multiple answers for a question you will be marked wrong. Answer the long answer questions directly on the exam. You must show your work for full credit. Answers may be left as fractions. Please place a box around final answers when appropriate. Good luck!

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**Name:**

**ID Number:**

**Section:**

### SECTION I: MULTIPLE CHOICE (60 points)

1. Jack's utility from chips (C) and salsa (S) is given by the function  $U(C,S)$ . If  $U(4,2) = 10$ ,  $U(2,4) = 10$  and Jack has convex preferences, we can say that:

- (a)  $U(3,3) \geq 10$ .
- (b)  $U(1,6) \geq 10$ .
- (c)  $U(3,3) \leq 10$ .
- (d)  $U(1,6) \leq 10$ .

(a) Notice that (3,3) is an average of the two bundles given in the problem. Since both of those bundles give a utility of 10 and Jack's preferences are convex, the average bundle must give Jack a utility of at least 10.

2. If bananas are an inferior, ordinary good, the the Engel curve for bananas is \_\_\_\_\_ and the demand curve for bananas is \_\_\_\_\_.

- (a) Upward sloping, downward sloping.
- (b) Upward sloping, upward sloping.
- (c) Downward sloping, upward sloping.
- (d) Downward sloping, downward sloping.

(d) Since bananas are an inferior good, a higher quantity of bananas will be consumed at lower income levels. This implies a downward sloping Engel curve. Since bananas are ordinary, demand will increase as price decreases producing a downward sloping demand curve.

3. Currently Jill is consuming positive amounts of books and magazines. At her current level of consumption, the marginal utility of a book is 4 utils and the marginal utility of a magazine is 3 utils. If the price of a magazine is \$1 and the price of a book is \$3, Jill can increase her utility by:

- (a) Buying more books and fewer magazines.
- (b) Buying more magazines and fewer books.
- (c) Buying fewer books and fewer magazines.

- (d) Not enough information.
- (b) If Jill buys one less book, she can buy three more magazines. The loss in utility from giving up a book would be 4 utils while the gain in utility from three more magazines would be 9 utils. Clearly, Jill can increase her utility by buying fewer books and more magazines.
4. Consider two utility functions  $U_A(x, y)$  and  $U_B(x, y)$ . If  $U_A(x, y) = -U_B(x, y)$ ,
- The two utility functions will represent the same preferences.
  - An indifference curve for  $U_A$  will have the same slope at any point  $(x, y)$  as an indifference curve for  $U_B$ .
  - The marginal rate of substitution for  $U_A(x, y)$  will be different than the marginal rate of substitution for  $U_B(x, y)$ .
  - None of the above.
- (b) The two utility functions will have the same marginal rate of substitution which will also mean that they have the same shape for their indifference curves. However, utility is increasing opposite directions for the two functions so they do not represent the same preferences.
5. If we have a graph with apples on the horizontal axis and oranges on the vertical axis, an increase in the price of oranges will:
- Make the budget line steeper.
  - Make the budget line flatter.
  - Make the indifference curves steeper.
  - Make the indifference curves flatter.
- (b) The magnitude of the slope of the budget line would be the price of apples divided by the price of oranges. If the price of oranges increases, the magnitude of the slope gets smaller.
6. If we like to consume cookies and milk together and both are normal, ordinary goods, we would expect the price offer curve to be:
- Upward sloping.
  - Downward sloping.
  - Horizontal.
  - The sign of the slope depends on which good is on the horizontal axis.
- (a) If we like to consume the two goods together, they are complements. As the price of one good decreases, the demand for that good will increase because the goods are ordinary while the demand for the other good will also increase because the goods are complements. This implies an upward sloping price offer curve.
7. If we have a graph with good  $x$  on the horizontal axis and good  $y$  on the vertical axis and the indifference curves are all vertical lines, we can say that:
- The marginal utility of good  $x$  is zero.
  - Goods  $x$  and  $y$  are perfect complements.
  - The marginal utility of good  $y$  is zero.

- (d) The marginal rate of substitution is diminishing.
- (c) Vertical indifference curves means that holding the value of good  $x$  constant, a change in the amount of good  $y$  consumed does not change utility. So the marginal utility of good  $y$  must be zero.
8. Suppose utility from coffee (C) and tea (T) is given by  $U(C, T) = C^2T^{\frac{1}{2}}$ . Which of the following statements are true?
- (a) The marginal utility from coffee is diminishing as the amount of coffee increases.
- (b) The marginal utility from tea is diminishing as the amount of tea increases.
- (c) (a) and (b) are both true.
- (d) Neither (a) nor (b) is true.
- (b) The marginal utility from coffee is  $MU_C = 2CT^{\frac{1}{2}}$ . This increases as  $C$  increases. The marginal utility from tea is  $MU_T = \frac{1}{2}C^2T^{-\frac{1}{2}}$ . This decreases as  $T$  decreases.
9. For a Giffen good, we can say for certain that the slope of the Engel curve and the slope of the demand curve:
- (a) Have the same signs.
- (b) Have opposite signs.
- (c) Get larger as the amount of the good increases.
- (d) Get smaller as the amount of the good increases.
- (b) All Giffen goods are inferior goods. So the slope of the Engel curve will be negative. The slope of the demand curve will be positive since the demand for a Giffen good increases when price increases.
10. Suppose that Bob has well-behaved, convex indifference curves and is currently maximizing his utility by consuming 5 donuts and 4 cups of coffee. If the price of donuts increases, what will be true of Bob's new optimal bundle of donuts and coffee:
- (a) It will contain more donuts than before and lie on a higher indifference curve.
- (b) It will contain more donuts than before and lie on a lower indifference curve.
- (c) It will contain fewer donuts than before and lie on a higher indifference curve.
- (d) It will contain fewer donuts than before and lie on a lower indifference curve.
- (d) If donuts get more expensive, then Bob will consume fewer donuts before and end up on a lower indifference curve (no point on his old indifference curve will be affordable after the price change). Note that the question wording was changed in the exam to specify that donuts are an ordinary good.
11. A doubling of the price of goods  $x$  and  $y$  and a tripling of income will:
- (a) Shift the budget line up and rotate it.
- (b) Shift the budget line down and rotate it.
- (c) Rotate the budget line but not shift it.
- (d) None of the above.
- (d) If the prices of both goods are doubled, then the slope of the budget line does not change so there is no rotation. However, income was increased by a larger factor than the prices, so the budget line would shift up.

12. Nancy's demand for apples is given by  $A = \frac{I}{p_A}$  where  $I$  is income and  $p_A$  is the price of apples. Suppose she spends all of her money on apples and bananas. When the price of apples decreases:
- The number of bananas consumed will increase.
  - The number of bananas consumed will decrease.
  - The number of bananas consumed will be unchanged.
  - The number of bananas consumed may increase or decrease.
- (c) From the demand function, it is clear that Nancy spends her entire income on apples. So no matter what the price of apples is, demand for bananas will always be zero.
13. Suppose that the price of muffins is reduced once you buy at least a dozen muffins. So each muffin up to a dozen costs one price and each muffin after that is sold at a lower price. On a graph with muffins on the horizontal axis and other goods on the vertical axis:
- The budget line will be kinked at twelve muffins and steeper to the right of the kink than to the left of it.
  - The budget line will be kinked at twelve muffins and steeper to the left of the kink than to the right of it.
  - The indifference curves will be kinked at twelve muffins.
  - The indifference curves will have a satiation point at twelve muffins.
- (b) For the region between zero muffins and 12 muffins, the slope of the budget line will be  $-\frac{p_{high}}{p_{other}}$  where  $p_{high}$  is the high price for muffins. The slope to the right of twelve muffins will be  $-\frac{p_{low}}{p_{other}}$  where  $p_{low}$  is the discounted price. Since  $p_{high} > p_{low}$ , the slope will be steeper to the left of twelve muffins.
14. If Hank's utility from goods  $x$  and  $y$  is described by the function  $U(x, y) = -2x + 3y$ , then:
- His indifference curves are straight lines with a slope of  $-\frac{2}{3}$ .
  - His indifference curves are straight lines with a slope of  $-\frac{3}{2}$ .
  - For certain ratios of positive prices, Hank will spend all of his money on  $x$ .
  - None of the above.
- (d) Notice that  $x$  is a bad while  $y$  is a good. This will lead to positively sloped indifference curves and lead Hank to spend all of his money on  $y$  for any ratio of positive prices.
15. On a graph of  $U(y)$  with  $y$  on the horizontal axis and utility on the vertical axis, the slope of the curve is equal to:
- The marginal rate of substitution.
  - The marginal utility of  $y$ .
  - The ratio of prices.
  - The marginal utility of  $x$ .
- (b) The slope of the curve is just the derivative of the utility function with respect to  $y$  which is the same thing as the marginal utility of  $y$ .

16. Demand for cupcakes (C) in terms of the price of cupcakes ( $p_C$ ), the price of sundaes ( $p_S$ ) and income ( $I$ ) is:

$$C = \frac{10I}{p_C + 10\frac{1}{p_S}}$$

Which of the following statements is true?

- (a) Cupcakes and sundaes are substitutes.
- (b) Cupcakes are an inferior good.
- (c) The Engel curve for cupcakes is downward sloping.
- (d) The demand curve for cupcakes is upward sloping.

(a) Looking at the demand equation, if  $p_S$  increases, the denominator gets smaller and demand for cupcakes will increase. So cupcakes and sundaes are substitutes. As  $I$  increases, demand for cupcakes increases so cupcakes are a normal good and would have an upward sloping Engel curve. As  $p_C$  increases, demand for cupcakes decreases, so the demand curve would be downward sloping.

17. If the marginal utility of hot dogs is diminishing, then we can say for certain that:

- (a) The utility from eating 10 hotdogs is less than the utility from eating 5 hotdogs.
- (b) The change in utility from eating the 10th hot dog will be smaller than the change in utility from eating the 5th hot dog.
- (c) A graph of utility as a function of hotdogs has an increasing slope.
- (d) A graph of utility as a function of hotdogs has a negative slope.

(b) The utility from eating 10 hotdogs will be more than the utility from eating 5 but the change in utility from eating the 10th hot dog will be smaller than the change in utility from eating the 5th hotdog.

18. If Spam is an inferior, ordinary good, an increase in income will \_\_\_\_\_ while an increase in the price of Spam will \_\_\_\_\_.

- (a) Increase the quantity of Spam consumed, increase the quantity of Spam consumed.
- (b) Increase the money spent on Spam, increase the quantity of Spam consumed.
- (c) Decrease the money spent on Spam, decrease the quantity of Spam consumed.
- (d) Increase the money spent on Spam, decrease the quantity of Spam consumed.

(c) If Spam is inferior, consumption will go down when income goes up. If prices are staying the same, this means that less money will be spent on Spam when income rises. If spam is ordinary, an increase in price will lead to a decrease in demand for Spam.

19. If two consumers with well-behaved, convex preferences face the same prices for goods  $x$  and  $y$  but have different incomes, what will definitely be true if both consumers maximize their utility?

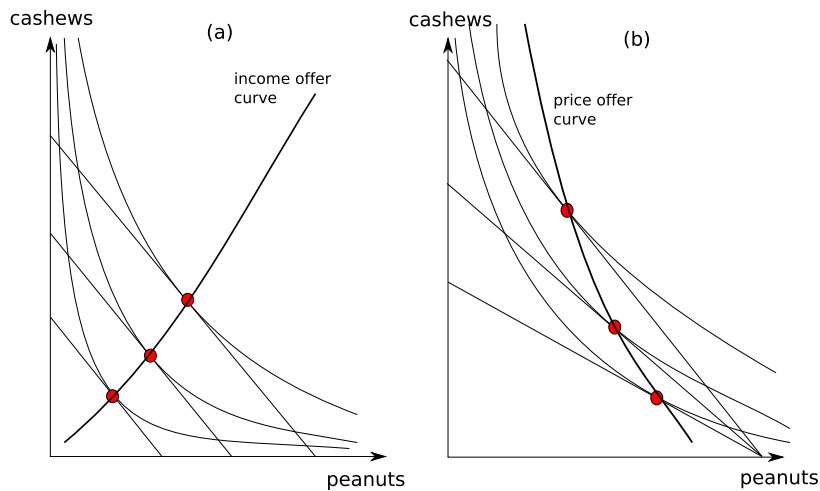
- (a) Both consumers will choose the same bundle.
- (b) Both consumers will have the same value for their marginal rate of substitution at their optimal bundle.

- (c) Both consumers will have the same value for their marginal utility from  $x$  and for their marginal utility from  $y$  at their optimal bundle.
  - (d) Both consumers will have the same utility level.
    - (b) Both consumers are still setting  $MRS$  equal to the ratio of the prices. If they face the same prices, they will reach the same  $MRS$ . However, if they have different incomes (or different preferences) they will wind up at different bundles with potentially different marginal utilities for  $x$  and  $y$ .
20. If the price of one good  $x$  increases and the price of good  $y$  decreases, a consumer's utility will:
- (a) Increase.
  - (b) Decrease.
  - (c) Stay the same.
  - (d) Not enough information.
    - (d) Whether the consumer's utility increases or decreases will depend both on how large the price increase for  $x$  is relative to the price decrease for  $y$  and on the consumer's indifference curves.

## SECTION II: SHORT ANSWER (40 points)

For this section, be certain to show your work and clearly label any graphs you draw. Give complete answers but keep them concise. Please place a box around final answers where appropriate.

- (6 points) Suppose that cashews are a normal, ordinary good, peanuts are a normal, ordinary good and peanuts and cashews are substitutes. On graph (a) below, show three points on the income offer curve. Include the budget lines and indifference curves that correspond to those three points. On graph (b) below, show three points on the price offer curve when the price of cashews is varied. Include the budget lines and indifference curves that correspond to those three points. Assume that the consumer has well-behaved, convex indifference curves.

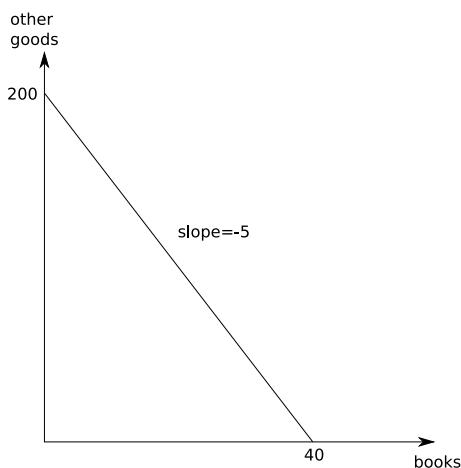


As income increases, the quantities of both goods should increase (since both are normal goods) giving us an upward sloping income offer curve. As the price of cashews increases, the quantity of cashews goes down because cashews are a normal good and the quantity of peanuts goes up because peanuts and cashews are substitutes. This gives us a downward sloping price offer curve.

2. (20 points) Suppose that your total income is \$200. Books cost \$5 each. Your utility from books ( $B$ ) and from all other goods ( $O$ ) is given by:

$$U(B, O) = 4B + 2O$$

- (a) Graph your budget constraint on a graph with books on the horizontal axis and other goods on the vertical axis. You can assume that the price of other goods,  $p_O$ , is \$1.



- (b) Derive expressions for the marginal utility of books ( $MU_B$ ), the marginal utility of other goods ( $MU_O$ ) and the marginal rate of substitution ( $MRS$ ).

$$MU_B = \frac{dU(B, O)}{dB} = 4$$

$$MU_O = \frac{dU(B, O)}{dO} = 2$$

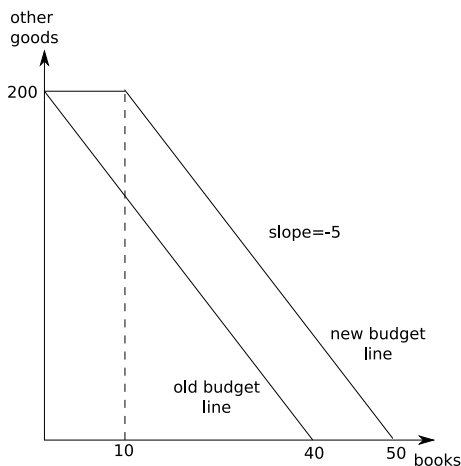
$$MRS = -\frac{MU_B}{MU_O} = -\frac{4}{2} = -2$$

- (c) Find the combination of books and other goods that maximizes your utility given your budget constraint. (You can consume fractions of books and fractions of other goods.)

Notice that the slope of the budget line is  $-5$  while the slope of the indifference curves is  $-2$ . Books cost five times as much as other goods but only give us two times as much utility. To maximize utility, we will spend all of our money on other goods. With \$200 to spend, the means 200 units of other goods and zero books.

- (d) Now suppose you are given a \$50 gift card for the bookstore. The gift card can only be spent on books. Graph your new budget constraint on a graph with books on the horizontal axis and other goods on the vertical axis.





If we are spending more than \$50 on books, our new budget constraint looks just like a regular budget constraint where the price of books is \$5, the price of other goods is \$1 and income is \$250. However, the gift card isn't quite the same as cash. We can't use it to buy other goods. So the most we can spend on other goods is still \$200. This gives us a flat budget line between (0 books, 200 other goods) and (10 books, 200 other goods).

- (e) Find your new optimal combination of books and other goods.

For spending our cash, books are still a bad option. So we will still spend all our \$200 on other goods, buying 200 units of other goods. However, we won't let the \$50 gift card go to waste. Since it can only be used on books, we will use it to buy 10 books. So our new optimal bundle is 200 units of other goods and 10 books.

- (f) By how much did the gift card increase your utility?

$$U(0, 200) = 4 \cdot 0 + 2 \cdot 200 = 400$$

$$U(10, 200) = 4 \cdot 10 + 2 \cdot 200 = 440$$

The gift card has increased our utility from 400 to 440. So utility has gone up by 40 units.

3. (14 points) Andy's utility from hours of tennis (T) and hours of golf (G) is given by:

$$U(T, G) = 2T^{\frac{2}{3}} + G^{\frac{2}{3}}$$

- (a) Derive expressions for  $MU_G$ ,  $MU_T$  and the  $MRS$ .

$$MU_G = \frac{dU(T, G)}{dG} = \frac{2}{3}G^{-\frac{1}{3}}$$

$$MU_T = \frac{2}{3}2T^{-\frac{1}{3}} = \frac{4}{3}T^{-\frac{1}{3}}$$

$$MRS = -\frac{MU_G}{MU_T} = -\frac{\frac{2}{3}G^{-\frac{1}{3}}}{\frac{4}{3}T^{-\frac{1}{3}}}$$

$$MRS = -\frac{1}{2} \left( \frac{T}{G} \right)^{\frac{1}{3}}$$

- (b) Derive an expression for demand for hours of tennis in terms of income ( $I$ ), the price of an hour of tennis ( $p_T$ ) and the price of an hour of golf ( $p_G$ ). In other words, derive the function  $T(p_T, p_G, I)$  that gives the optimal number of hours of tennis for any set of prices and income.

Start with the tangency condition by setting the slope of the budget line equal to the slope of the indifference curve:

$$-\frac{p_G}{p_T} = MRS$$

$$-\frac{p_G}{p_T} = -\frac{1}{2} \left( \frac{T}{G} \right)^{\frac{1}{3}}$$

$$\left( \frac{2p_G}{p_T} \right)^3 = \frac{T}{G}$$

$$G = \left( \frac{p_T}{2p_G} \right)^3 T$$

Plug this result into the budget equation for  $G$  and solve for  $T$ :

$$p_G G + p_T T = I$$

$$p_G \left( \frac{p_T}{2p_G} \right)^3 T + p_T T = I$$

$$\left( \frac{p_T^3}{8p_G^2} + p_T \right) T = I$$

$$T = \frac{I}{\frac{p_T^3}{8p_G^2} + p_T}$$

- (c) Based on your expression in part (b), determine whether hours of tennis are a normal or inferior good, whether they are an ordinary or Giffen good, and whether tennis and golf are substitutes.

From the demand equation, we can see that if income increases,  $T$  increases so hours of tennis are a normal good. When  $p_T$  increases, the denominator gets larger in the demand function, decreasing  $T$ . So hours of tennis are an ordinary good. Finally, when  $p_G$  increases, the denominator gets smaller leading to an increase in  $T$ . So hours of tennis and hours of golf are substitutes.